To Forecast or Not to Forecast: An Application to Nonlinear Models

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2. Forecasting with linear models is straightforward; no error distribution assumptions required.
3. Thus, is it possible to approximate nonlinear predictions with linear ones? When not, what approach should one use: direct or iterated forecasts?
4. No research (that we are aware of) yet on this issue.

Contribution

1. Propose a pre-testing for nonlinearity approach to forecasting with non-linear models; extend the linearity test of Terasvirta (1994) to various forecasting horizons; thus, focus on smooth transition autoregressive (i.e., STAR) models.

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3. Findings: (a) when the (non)linearity test strongly rejects the null of linearity, then a nonlinear model clearly outperforms a linear model;
4. (b) when it fails to reject the null, a logistic and a simple AR model display similar performance; (c) for nonlinear models: the "direct" method performs better than the bootstrap predictor at shorter forecast horizons, but the evidence is mixed at longer horizons.

Previous Research

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3. Marcellino et al. (2006) - large empirical study using several competing AR models and find that multi-step iterated forecasts are more accurate than "direct" ones
4. Leybourne et al. (1998), Sollis et al. (2002), Kapetanios et al. (2003) - OECD countries RER display some type of smooth adjustment, either asymmetric or asymmetric.

The model

1. Follow Terasvirta (1994) - use a third order Taylor series approximation of the STAR component
2. A univariate STAR model of order 1:

$$y_{t+1} = \alpha w_t + \beta w_t G(\theta; y_{t-d}; c) + \epsilon_{t+1}, t = 1, \ldots, T \quad (1)$$

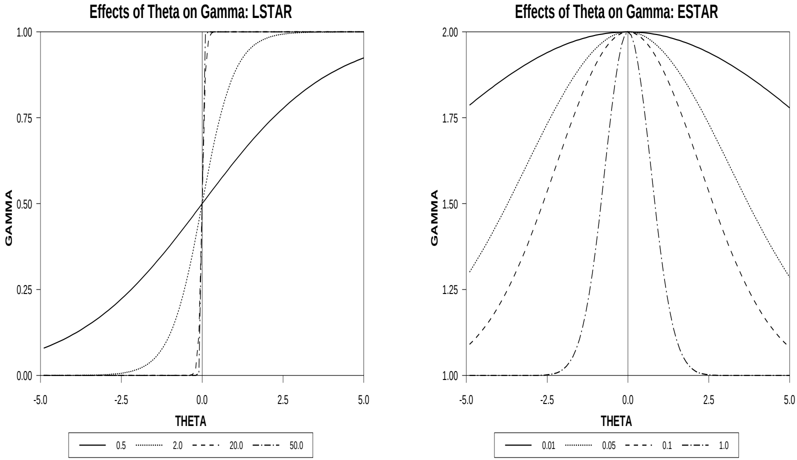
   where $w_t = (y_{t-k}, ..., y_{t-p})$ and $0 \leq k \leq p$. Logistic function:

$$G(\theta; y_{t-d}, c) = [1 + exp(-\theta(y_{t-d} - c))]^{-1}, \quad (2)$$

   where $\theta > 0$, $d \geq 1$ is the delay parameter, and c is the location parameter (i.e. threshold); θ is the slope parameter.
3. Exponential function:

$$G(\theta; y_{t-d}, c) = 1 - exp(-\theta(y_{t-d} - c)^2), \quad (3)$$

LSTAR vs. ESTAR



The models

1. Remarks: Whenever $|y_{t-d} - c|$ is large and $y_{t-d} < c$, y_t is effectively generated by the linear model:

$$y_t = \alpha y_{t-1} + \epsilon_t, t = 1, \ldots, T. \tag{4}$$

   If $|y_{t-d} - c|$ is large and $y_{t-d} > c$, y_t is virtually generated by:

$$y_t = (\alpha + \beta) y_{t-1} + \epsilon_t, t = 1, \ldots, T. \tag{5}$$

2. Reparameterize to get:

$$\Delta y_t = \delta y_{t-1} + \beta y_{t-1} [1 + exp(-\theta(y_{t-d} - c))]^{-1} + \epsilon_t, \tag{6}$$

   where $\delta = \alpha - 1$. When $\theta = 0 \Rightarrow$ the transition function $G(\theta; y_{t-d}; c) \equiv 1/2$ so that the LSTAR model nests a linear model. Conversely, when $\theta \to \infty$ the LSTAR model - switching regime with two distinct regimes.

3. The ESTAR process: bounded between 0 (i.e., $\theta = 0$) and 1 (i.e., $\theta \to \infty$).

Forecasting with an AR process

1. Forecasting with linear models is straightforward. For an AR(1):

$$y_{t+1} = \delta + \rho w_t + \epsilon_{t+1} = \delta + \rho \sum_{k=0}^{p} \rho_k y_{t-k} + \epsilon_{t+1}. \quad (7)$$

   an iterated forecast is obtained recursively as:

2. $$\hat{y}^I_{t+h|t} = \hat{\delta} + \hat{\rho} \sum_{k=0}^{p} \hat{\rho}_k \hat{y}^I_{t+h-1-k|t}. \quad (8)$$

3. In contrast, a "direct" forecast writes as:

$$\hat{y}^D_{t+h|t} = \hat{\delta} + \hat{\rho} \sum_{k=0}^{p} \hat{\rho}_k y^D_{t-k}. \quad (9)$$

   Adjust the forecasts if the series needs to be first-differenced.

Forecasting with a STAR process

1. Predictions from a STAR process write as:

$$y_{t+h} = E(y_{t+h}|w_t) = \int_{-\infty}^{+\infty} f(y_{t+h}|w_{t+1}) f(w_{t+1}|w_t) dw_{t+1} \qquad (10)$$

2. Need assumptions about the error distribution $G(\epsilon_{t+1}, ..., \epsilon t + h)$; generally, use a Monte Carlo simulation or bootstrap the residuals.
3. The steps above require that the nonlinear model be correctly specified $\rightarrow$ the "direct" method more robust to model misspecification.
4. Use Terasvirta's (1994) approach to test for linearity at the desired forecast horizon $t + h$: approximate the nonlinear component through Taylor series expansion (due to the identification issues).

Forecasts from LSTAR and ESTAR processes

1. Simplify the LSTAR model to:

$$y_{t+1} = \alpha y_t + \beta y_t \frac{1}{1 + exp(-\theta(y_{t-d} - c))} + \epsilon_{t+1}, t = 1, \ldots, T \quad (11)$$

and let $z = \theta(y_t - c)$ s.t. $G = [1 + exp(-z)]^{-1}$.

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   and let $z = \theta(y_t - c)$ s.t. $G = [1 + exp(-z)]^{-1}$.

2. Expand G around z up to the third order and evaluate the expression at $z = 0$ for various forecasting horizons. Example:

$$y_{t+2} : \frac{1}{4}\beta^2 y_t + \frac{1}{4}\beta^2 y_t z + \frac{1}{16}\beta^2 y_t z^2 + \frac{1}{48} y_t z^3 + \epsilon_{t+2}$$

   up to :

$$y_{t+12} : \frac{1}{4096}\beta^{12} y_t + \frac{3}{2048}\beta^{12} y_t z + \frac{33}{8192}\beta^{12} y_t z^2 + \frac{27}{4096} y_t z^3 + \epsilon_{t+12}$$

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3. For an ESTAR model we have that:

$$y_{t+k} = (\alpha^k + k\alpha^{k-1}\beta\theta c^2) y_t - (2k)\alpha^{k-1}\beta\theta c y_t^2 + k\alpha^{k-1}\beta\theta y_t^3 + \epsilon_{t+k}$$

A (Non)Linear Forecasting Test

1. The test with an LSTAR under the alternative writes as:

$$y_{t+h} = \alpha' w_t + \beta_1'(w_t y_{t-d}) + \beta_2'(w_t y_{t-d}^2) + \beta_3'(w_t y_{t-d}^3) + \epsilon_{t+h} \quad (12)$$

   and the null hypothesis is: $\beta_1' = \beta_2' = \beta_3' = 0$. The asymptotic distribution: $LM \sim \chi[3(p-k+1)]$; or allow the lag order p to depend on the forecasting horizon $\rightarrow LM \sim \chi[3(p_k - k + 1)]$.
2. The degrees of freedom represent the number of variates in the nonlinear approximation
3. For an ESTAR, the null writes as: $\beta_1' = \beta_2' = 0$ and $LM \approx \chi[2(p-k+1)]$
4. Perform the LM tests as F-tests for small samples.

Small Sample Properties: Size simulations

1. Size simulations:
$$y_t = \rho y_{t-1} + \epsilon_t \tag{13}$$
   where $\rho \in \{0.1, 0.3, 0.5, 0.8\}$.
2. Size performance - 20,000 replications, 5% nominal size, sample sizes of $T \in \{50, 100, 200\}$, at 12 forecast horizons;
3. Two cases: Case 1 uses 2 terms 2 in the Taylor series expansion; Case 2 uses 3 terms.
4. Findings: (a) good size for low ρ's, over-reject for higher ones; (b) the degree of over-rejection increases with higher h; more so in Case 1 than on Case 2.

Small Sample Properties: Power simulations

1. LSTAR:

$$y_{t+1} = 0.8y_t - \beta[1 + exp(-\theta(y_t - c))]^{-1} + \epsilon_{t+1} \qquad (14)$$

2. ESTAR:

$$y_{t+1} = 0.8y_t - \beta[1 - exp(-\theta(y_t - c)^2)] + \epsilon_{t+1} \qquad (15)$$

   where $\beta \in \{0.1, 0.3, 0.5, 0.8\}$, $\theta \in \{0.5, 2, 20, 50\}$ for LSTAR and $\theta \in \{0.01, 0.05, 0.1, 1.0\}$ for ESTAR, and $c \in \{0,1.0\}$. Use one, four, eight, and twelve forecasting horizons.

3. Case 1: set c $=$ 0 and use a 3rd order approximation; Case 2: c $\neq$ 0 and use a 3rd order approximation; Case 3: c c $\neq$ 0 and use a 2nd order approximation.
4. Findings: good power for high values of β and θ; when $\beta \leq 0.5$ and fixed θ, power increases with the forecasting horizon; vice-versa for β $>$ 0.5 and fixed θ
5. For fixed β and θ, power increases with the sample size;

Data Description

1. Apply the test to a set of bilateral RER and real effective exchange rates (REERs) from several OECD countries; use both the CPI and PPI to compute the RER.
2. Data source: International Financial Statistics (IFS); use quarterly data: January 1957 - April 2007. This period comprises the fixed exchange rate period (1957-1971) and the flexible exchange rate period (January 1973 - April 2007).
3. Apply the linearity test at one, four, eight, and twelve forecasting horizons, respectively; use a 3rd order approximation (i.e., lower size distortion, higher power for larger sample sizes).

Some results

Table: Nonlinearity Tests for Bilateral and Real Effective Exchange Rates: use 3 terms

	RER(PPI/WPI)				RER(CPI)				REER			
Steps ahead	One	Four	Eight	Twelve	One	Four	Eight	Twelve	One	Four	Eight	Twelve
Australia												
1957:1-2007:1	0.76	0.60	0.06	0.91	2.51*	0.62	2.67**	1.09	3.67**	0.91	3.58**	1.79
Austria												
1957:1-1998:4	3.11**	15.03***	0.83	22.37***	3.12**	15.07***	0.83	22.36	1.02	5.51***	1.88	1.41
Belgium												
1957:1-1998:4	0.72	26.51***	3.41	0.44	24.31***	5.15***	2.92**	2.70**	7.23***	5.63**	2.30*	6.12***
Canada												
1957:1-2007:1	2.58*	0.83	1.17	1.62	2.27*	2.61*	0.34	2.81*	2.91*	0.78	0.51	2.43*
Denmark												
1957:1-2007:1	0.42	0.15	0.80	0.42	0.22	0.04	2.08	0.94	9.23***	1.42	3.37**	0.07
Finland												
1957:1-1998:4	1.70	2.49*	23.56***	14.35***	1.68	2.48*	23.56***	14.35***	3.38**	2.30*	0.35	0.61
France												
1957:1-1998:4	-	-	-	-	11.60***	16.41***	13.22***	9.21***	1.25	0.61	0.36	0.39
Germany												
1957:1-1998:4	-	-	-	-	6.17***	1.52	13.27***	4.74***	1.58	0.30	0.99	1.63
Greece												
1957:1-2000:4	7.39***	9.74***	14.82****	13.27***	7.22***	9.30***	14.40***	12.23***	0.50	1.12	0.17	1.33
Ireland												
1957:1-1998:4	18.16***	33.00***	17.22***	27.78***	18.19***	33.58***	17.29***	27.79***	-	-	-	-
Italy												
1957:1-1998:4	17.59***	35.38***	17.26***	27.76***	17.65***	35.38***	17.15***	27.56***	2.53*	8.45***	6.39***	1.14

More Results

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	RER(PPI/WPI)				RER(CPI)				REER			
Steps ahead	One	Four	Eight	Twelve	One	Four	Eight	Twelve	One	Four	Eight	Twelve
Japan												
1957:1-2007:1	0.25	0.49	0.28	1.22	0.90	0.38	0.39	1.70	6.85^{***}	0.20	2.11	0.52
Luxembourg												
1957:1-1998:4	2.40^{*}	1.45	0.98	0.06	0.82	5.77^{***}	9.89^{***}	3.80^{**}	4.02^{***}	2.79^{**}	0.99	1.06
Netherlands												
1957:1-1998:4	2.17^{*}	16.52^{***}	15.49^{***}	24.06^{***}	2.19^{*}	16.54^{***}	15.55^{***}	23.40^{***}	2.30^{*}	1.18	11.92^{***}	4.38^{***}
Norway												
1957:1-2007:1	11.53^{***}	1.04	0.05	0.00	0.05	0.04	0.77	2.05	0.39	1.20	0.88	0.75
New Zealand												
1957:1-2007:1	1.68	0.23	0.41	6.19^{***}	0.46	2.51	0.82	0.44	0.32	4.55	0.61	1.93
Spain												
1957:1-1998:4	1.89	2.46^{*}	3.83^{*}	20.94^{***}	1.90	2.47^{*}	3.82^{**}	20.94^{***}	3.31^{**}	3.41^{**}	0.43	1.53
Sweden												
1957:1-2007:1	1.61	0.04	0.19	0.37	1.06	0.36	0.04	2.56	0.06	4.89^{***}	0.17	0.08
Switzerland												
1957:1-2007:1	1.64	0.14	0.18	0.58	1.54	0.67	1.42	0.98	2.51^{*}	7.71^{***}	1.52	0.39
UK												
1957:1-2007:1	2.29^{*}	2.67^{**}	0.47	0.74	0.48	1.12	0.40	1.03	1.41	1.91	1.12	1.76

* Significance at the 10% level; ** Significance at the 5% level; *** Significance at the 1% level

Empirical Findings

1. More evidence of nonlinear patterns for CPI and PPI-based RER than for REERs.
2. A series may require a linear/nonlinear model depending on the forecasting horizon.
3. All Euro area OECD countries display some degree of nonlinear behavior at various horizons, while non-Euro (e.g., Denmark, Norway, Sweden, Switzerland, and the UK) area ones do not.
4. In general, whenever the null is rejected, the evidence suggests that most countries display LSTAR-type mean reversion to long-run equilibrium.

Empirical Findings: Direct vs. Recursive Models

1. Marcellino et al. (2006) compare several AR models: recursive forecasts outperform direct ones;
2. This study limits to STAR-type models. Consider: Australia's PPI based RER (at 4 and 12 steps ahead), France's CPI based RER, Italy's PPI based RER, Japan's CPI based RER, Netherlands's REER (all 12 steps ahead), and Norway's CPI based RER at 4 steps ahead.
3. Failed to reject the null for Australia's and Japan's RER, and Norway's REER, respectively; the rest appear nonlinear.
4. 4 models: fixed AR(4), fixed AR(12), AR(BIC), AR(AIC). Use 3 criteria: mean error, MSPE, MAE.

Direct vs Iterated: Linear Case

Table: Direct vs. Iterated Forecasting Methods: A Linear Approach

	AR(4)			AR(12)			AR(BIC)			AR(AIC)		
Criterion	Mean	MSPE	MAE	Mean	MSPE	MAE	Mean	MSPE	MAE	Mean	MSPE	MAE
Australia:	4 steps											
Iterated	-0.04941	0.00504	0.06820	-0.03670	0.00436	0.06397	-0.03566	0.00316	0.05502	-0.03733	0.00435	0.06460
Direct	-0.29684	0.12057	0.31483	-0.26908	0.10071	0.28707	-0.29611	0.11900	0.31409	-0.27178	0.10298	0.28977
Australia:	12 steps											
Iterated	0.04247	0.00798	0.04994	0.05771	0.00869	0.05834	0.04152	0.00786	0.04845	0.05771	0.00869	0.05834
Direct	-0.22902	0.06004	0.22902	-0.22512	0.05769	0.22512	-0.22659	0.06009	0.22659	-0.22785	0.05868	0.22785
France:	12 steps											
Iterated	0.27658	0.21508	0.27658	0.29003	0.21941	0.29003	0.27078	0.21307	0.27078	0.29003	0.21941	0.29003
Direct	0.06230	0.01663	0.10834	0.06256	0.01644	0.10815	0.06245	0.01671	0.10860	0.06245	0.01671	0.10860
Italy:	12 steps											
Iterated	0.64279	4.21620	0.64612	0.68070	4.22331	0.68070	0.63929	4.21539	0.64272	0.68070	4.22331	0.68070
Direct	0.00563	0.00448	0.05405	-0.00891	0.00738	0.06870	0.00447	0.00301	0.04390	-0.00104	0.00736	0.06741
Japan:	12 steps											
Iterated	0.49669	1.78013	0.49669	0.54181	1.80608	0.54181	0.49502	1.77925	0.49502	0.50695	1.78277	0.50695
Direct	0.03011	0.00868	0.07333	0.02997	0.00868	0.07382	0.03011	0.00868	0.07333	0.02997	0.00868	0.07382
Netherlands:	12 steps											
Iterated	0.36924	1.71966	0.37311	0.40022	1.72143	0.40074	0.38178	1.72002	0.38225	0.40022	1.72143	0.40074
Direct	0.05988	0.00478	0.05988	0.05722	0.00443	0.05722	0.06110	0.00489	0.06110	0.05722	0.00443	0.06105
Norway:	4 steps											
Iterated	0.34010	0.75999	0.43808	0.34506	0.75952	0.43311	0.35161	0.75877	0.42656	0.35379	0.75874	0.42439
Direct	-1.55343	3.07805	1.58368	-1.54672	3.05162	1.57696	-1.55423	3.08090	1.58448	-1.54735	3.05405	1.57759

Mean - mean of forecasts; MSPE - mean square predicted error; MAE - absolute mean of forecasts

Empirical Findings: Direct vs. Recursive Models

1. MSPE increases with the forecast horizon;
2. When the test strongly rejects the null, the direct method clearly dominates (e.g., Italy, Netherlands) $\longrightarrow$ strengthen the theoretical finding that the direct method fares better when model is misspecified.
3. Evidence is mixed when the model appears linear.

Direct vs Iterated: Nonlinear Case

Table: Direct vs. Iterated Forecasting Methods: A Nonlinear Approach

	Monte Carlo			Bootstrap			Direct		
Criterion	Mean	MSPE	MAE	Mean	MSPE	MAE	Mean	MSPE	MAE
Australia:	4 steps								
Criterion	-0.05606	0.00329	0.05606	-0.05344	0.00299	0.05344	-0.04496	0.00253	0.04496
Australia:	12 steps								
Criterion	0.00309	0.00173	0.03373	0.00356	0.00170	0.03345	-0.07653	0.00744	0.04431
France:	12 steps								
Criterion	0.09691	0.01316	0.09691	0.10044	0.01403	0.10044	0.01611	0.00434	0.06196
Italy:	12 steps								
Criterion	0.02264	0.00226	0.04232	0.06086	0.00663	0.06642	0.12825	0.02127	0.07381
Japan:	12 steps								
Criterion	0.09860	0.01412	0.09860	0.10602	0.01609	0.10602	0.02704	0.00784	0.06204
Netherlands:	12 steps								
Criterion	0.00872	0.00015	0.00954	0.02123	0.00065	0.02163	0.06482	0.00432	0.05926
Norway:	4 steps								
Criterion	-0.04136	0.00181	0.04136	-0.04153	0.00182	0.04153	-0.02446	0.00076	0.02446

Mean - mean of forecasts; MSPE - mean square predicted error; MAE - absolute mean of forecasts

Empirical Findings: Direct vs. Recursive Models

1. Use both the Monte Carlo and bootstrap approach: no clear winner
2. At shorter horizons the direct method appears to have a slight advantage
3. When the test strongly rejects the null, the bootstrap appears to dominate the direct approach (i.e., Italy and Netherlands vs. France).
4. Comparing the linear and nonlinear results, it appears that an LSTAR model dominates when the test strongly rejects the null; similar performance when models are approximately linear.

Conclusion

1. Pretesting for linearity before forecasting appears to be useful; the test has good size for the less persistent ARs and good power at the longer horizons;
2. OECD Euro area countries display higher non-linear dynamics of their RER than non-Euro area ones $\longrightarrow$ further research?; REERs appear linear relative to bilateral RERs.
3. When the test strongly rejects the null of linearity $\Rightarrow$ a nonlinear model clearly dominates a linear one; the bootstrap appears to perform better than the direct approach for nonlinear series (evidence is mixed though).
4. the direct method performs better at shorter horizons.