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ORDERED MEAN DIFFERENCE
AND
STOCHASTIC DOMINANCE
AS
PORTFOLIO PERFORMANCE MEASURES

with an approach to cointegration

– ***dissertation paper*** –

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Bucharest
June 2002

CONTENTS

1. INTRODUCTION

❑ 1.1. PURPOSE

Measuring mutual fund performance is a topic of interest among the practitioners as well as for the theoreticians, especially since the mutual fund industry burst in the United States, in the 1960's. This is the time since when the first attempts to model the evolution of mutual funds portfolio were made. Ever since then the market timing ability of the fund managers appeared to be the most relevant determinant of fund performance. That is the topic that the greatest part of the studies tried to address. Some of the classical measures of fund performance will be briefly presented here in connection with a presentation of the concept of market timing.

This paper pursues the Bowden (2000) article, which introduces the measure of *ordered mean difference* (OMD) as a portfolio performance measure applicable to all risk averse investors, no matter what is his degree of aversion to risk. The OMD function can be obtained by both parametric and non-parametric procedures and represents "a function whose value at any point R is equal to the running mean difference between the fund return and the benchmark return" (as pointed out in Bowden (2000)).

As it will be shown, there is a tight connection between OMD and stochastic dominance. The utility function used to construct the OMD measure has the form of the payoff of a put option for a short position, as indicated in Merton (1981) and Henriksson and Merton (1981), and this represents exactly the opposite of the *poverty gap function*, as defined in Davidson and Duclos (2000). The poverty gap function appears in the context of the stochastic dominance. Bowden (2000) shows that the OMD construct can, relatively easily, be used for testing for second order stochastic dominance (SSD).

The choice of this type of utility function is motivated in literature by arguments that will be discussed in this paper. The idea comes from a Robert C. Merton's article (1981), where it is shown that there is a correspondence between certain option strategies, namely a protective put strategy, and the strategy used by a market timer fund manager, and so, the return on the market timing performer fund can be expressed as the return on a portfolio that used such a strategy.

This paper is structured in three parts: the first part is the introduction, the second lights the theoretical background needed for understanding the OMD, and the final, the third part, is the empirical one.

After presenting in the first part the concept of market timing and some of the most known classical measures of portfolio performance (Treynor-Mazuy, Jensen, Sharpe), which have the merit of introducing the risk in the context of measuring mutual fund performances, the second part introduces, in Section 2.1., the concept of equivalent margin, which is the fundament and the starting point in construction of the OMD function. Section 2.2. describes the utility function and the motivation for its usage. A connection with the stochastic dominance theory will be made in order to identify the similarity between the utility function and the poverty gap function and shows how this similarity can be interpreted. There is a deeper connection that will be made, relating the definitions of risk employed by Rotschild and Stiglitz (1970) and their equivalence. This equivalence will re-appear later, in Section 2.4., where it will be presented how the OMD function can be used a statistic for testing for second order stochastic dominance. Before that, section 2.3. is dedicated to constructing the OMD schedule. This section is probably the core of the paper. It is also shown here that the OMD formula can be used to identify how aggressive or defensive a fund has been. It will be shown that the equivalent margin is a weighted sum of OMD schedules, which means that any risk averse investor can be seen as a spectrum elementary investors, each of them having a put option profile utility function that differ from each other by the exercise price (or "focal point", as it will be called). The econometric model is constructed in Section 2.5., on the theoretical background presented all across the second part.

The third part employs the econometric model for a parametric estimation of OMD and reports the results obtain for a Romanian mutual fund. It is verified if the fund has been OMD dominant over the market and/or stochastically dominant. The final section of the paper connects the OMD principle with the econometric concept of cointegration between time series. The approach is motivated by the fact that both the OMD and the cointegration analysis are long term approaches.

1.2. Classical Measures of Portfolio Performance

By market timing one can understand the ability of a fund manager to predict the changes in the market and to react accordingly. Thus, in terms of the CAPM model, the fund manager would change the composition of the portfolio such as to increase its beta if he expects a bull market, and decrease the beta in anticipation of a bear market, such that the rising in the fund's return would be greater than the rising in the benchmark in the situation of a bull market, and the decreasing of the fund's return would not be more than the decreasing of the benchmark when bear market. Or, when the portfolio is a fixed-income one, the fund manager will try to modify the portfolio structure so as to increase the duration of the portfolio when he expects a falling in interest rates and vice-versa.

Henriksson and Merton (1981) make a distinction between "macroforecasting" and "microforecasting". By microforecasting they understand the "forecasting of price movements of individual stocks relative to stocks generally," while "macroforecasting" is "forecasting price movements of the general stock market relative to fixed income securities". Only the latter Henriksson and Merton refer to as "market timing," while the former is called "security analysis".

Merton (1981) developed a method for testing for market timing. One can find reminiscences of this methodology in Bowden's (2001) article that constitutes the starting point of the present paper. The connection will be presented in Section 2.

There have been developed several measures of measuring market timing ability and performance of mutual fund portfolios.

The ***Treynor measure*** expresses the portfolio's excess return per unit of risk, assuming complete diversification of the portfolio.

$$T = \frac{R_i - RFR}{\beta_i},$$ where:

- R_i: average rate of return for the portfolio during a specified period of time
- RFR: average rate of return of a risk-free investment during the same period of time
- β_i: the slope of the security market line (the portfolio's beta coefficient).

The larger the value of *T*, the more preferable for all investors the fund is. *T* itself can be regarded as a slope, respectively the slope of the line connecting the fund performance with the risk free rate. We can say that *T* is the equivalent of the difference between the market return and the risk free rate in the SML equation.

Another common measure used to measure the performance of a portfolio has been the ***Jensen's Alpha***, which is defined as the intercept term in an equation based on CAPM:

$R_{it} - RFR_t = \alpha_i + \beta(R_{Mt} - RFR_t) + \varepsilon_{it}$, where:

R_{it}: the return on the portfolio i at time t

RFR_t: risk free interest rate at time t

β_i: the systematic risk of the portfolio

R_{Mt}: the return on the market portfolio

α: the intercept, that shows whether the portfolio manager has superior ($\alpha>0$) or inferior ($\alpha<0$) market timing ability. (An α that is not significantly different from 0 means that the manager conducted a naïve buy and hold policy).

ε_{it}: the disturbance term

The Jensen's α reflects the part of the return attributable to the manager's market timing ability. The Jensen's measure is different from the other measures by the fact that requires a different risk free rate for each period of time. The Jensen's α represents the difference between the fund return and the return of a portfolio on the securities market line with the same β. If this difference is positive (that is, $\alpha > 0$), the line of the fund return lies above the SML, and the fund is considered to have superior performance relative to the market. Like the Treynor's measure, the Jensen's α presumes a completely diversified portfolio, because the risk premiums are given by β, which is the systematic risk.

Also derived from the CAPM and very close to the Treynor measure is the ***Sharpe Ratio***:

$S = \frac{R_i - RFR}{\sigma_i}$, where:

R_i: the rate of return for the portfolio

RFR: the risk free interest rate

$\sigma_{i:}$ the standard deviation of the return on the portfolio during a period of time.

Apart from the Treynor's and Jensen's measures, Sharpe uses the standard deviation of the return of the fund instead the beta coefficient as the measure of risk. This means that the presumption of well diversification does not hold in Sharpe's measure, and the ability of the fund manager to diversify the portfolio can also be checked. If the portfolio is perfectly diversified, the Sharpe ratio will equal the Treynor ratio.

An alternative approach to performance evaluation measurement is furnished in Chrétien and Ahn (2000). They pointed out the trade-off between precision and incorrect inference in performance measurement that is inherent in an analysis of a portfolio performance. All the studies presented above and the one that we will use in the empirical part of this paper have in view to obtain a maximal precision in the performance measurement and produce a point estimate of the performance measure. The trade-off is that they have to make auxiliary assumptions, and the so-called "bad model problem" may appear from this fact.

In their paper, Chrétien and Ahn follow the alternative path of the trade-off: they construct performance evaluation bounds, which give the admissible range of performance measure values a mutual fund is allowed to have, instead of making a point estimate. Proceeding this way, they lose in the precision of the evaluation, but avoid the "bad model problem". The performance bounds they construct can then be used to compare alternative performance measures suggested in the literature: if the candidate performance measure falls between the lower and the upper bound, it is admissible, since, by construction, the Chrétien and Ahn range includes the entire set of admissible measures.

Chrétien and Ahn (2000) develop three alternative ranking rules for mutual funds: strong form dominance, semi-strong form dominance and weak form dominance. In what follows, we will use the notation and definitions provided by the authors.

They consider two funds: A and B, and the corresponding performance bounds are: $PM_A = \left[\underline{\alpha}(x_A), \overline{\alpha}(x_A)\right]$ and $PM_B = \left[\underline{\alpha}(x_B), \overline{\alpha}(x_B)\right]$ respectively. The definitions of strong, semi-strong and weak form dominance are:

- *Strong form dominance*: Fund A dominates fund B in the sense of strong form dominance, denoted by $A \overset{SD}{>} B$, if the lower bound on the performance measure of A is greater than the upper bound on the performance measure of B: i.e., $\underline{\alpha}(x_A) > \overline{\alpha}(x_B)$.
- *Semi-strong form dominance*: Fund A dominates fund B in the sense of semi-strong form dominance, denoted by $A \overset{SSD}{>} B$, if the lower bound on the differential in performance measures of A and B is positive.
- *Weak form dominance*: Fund A dominates fund B in the sense of weak form dominance, denoted by $A \overset{WD}{>} B$, if the lower and upper bounds on the performance measure of A are greater than the lower and upper bounds on the performance measure of B respectively: i.e., $\underline{\alpha}(x_A) > \underline{\alpha}(x_B)$ and $\overline{\alpha}(x_A) > \overline{\alpha}(x_B)$.

2. CONSTRUCTION OF THE MODEL

2.1. THE EQUIVALENT MARGIN

The problem with those classical measures is that they depend upon the degree of risk aversion of a particular investor. They do not constitute a general-applicable measure of performance for all investors. The same results will be interpreted distinctly by a two investors with different degrees of risk aversion. This means that a fitted regression by itself does not automatically constitute a formal performance assessment in terms of any investor's welfare criterion.

The ordered mean difference (OMD) represents, as introduced in Bowden (2000), the function whose value at any point R is equal to the running (progressive) mean difference between the fund return (r) and the benchmark return (R) up to that point, where the observation are ordered by the benchmark return. Graphically, the OMD represents the area between the regression curve of the fund return on the benchmark return and the benchmark itself, on the abscissa. So, in order to measure by OMD the performance of a given fund, one will have to order the time series by the benchmark return and to calculate the mean difference between the fund return and the benchmark return at every point R. If this difference is always positive it mean that the fund OMD-dominated the market over the period of analysis, so that the investors had a surplus by investing in the fund.

The roots and the connections of the OMD criterion are multiple. At its base sits the notion of *equivalent margin*, which is borrowed from welfare economics and is used to formulate the definition of the investor's surplus.

Let us consider R the return on the benchmark and r the return on the fund we want to compute the OMD for. As benchmark, a market index is usually taken into account, but we one can take another fund's return as R, if a comparison between two funds is desired. We consider an investor who wants to form a portfolio from *x* units of the fund and *1-x* units of the benchmark. Taking the premise that the investor is risk averse, so he has a von Neumann – Morgenstern utility function, his problem consists in maximisation of his utility function $E_{r,R}U\left(x \cdot r + (1-x) \cdot R\right)$.

We consider further that a tax *t* is levied to penalise the return on the fund, r. The investor's decision problem becomes, in this case:

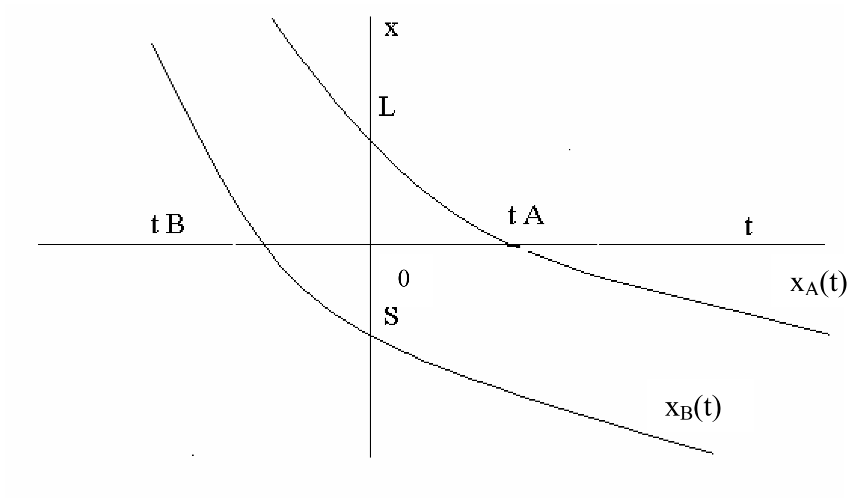
$$\max E_{r,R}U\left(x(r-t) + (1-x)R\right).$$

We can think of the tax *t* as a rent margin generated by the manager's market timing ability. Supposing that an investor is long in fund ($x>0$), the tax that is necessary to drive the holding to zero represents the equivalent margin. The higher the *t*, the more valuable the fund is considered to be. If the investor is short in the fund ($x<0$), the tax (or margin) *t* represents how much the investor has to be compensated to hold the fund. So, a positive *t* means that the fund has value for the investor if he holds long the fund, and if t is less than 0, the fund's superiority is derived from going short. Thus, this welfare measure (welfare gains or losses) is itself a rate of return.

This criterion can be regarded as a measure relating to what in Bowden (2001) is called *relative efficiency*. A benchmark portfolio R is said to be efficient relative to fund (or security) r if there is no utility value in adding further units of r to the benchmark portfolio R; if there is utility value from going long or short in r, the existing benchmark portfolio is not efficient for the investor. If the manager can add value by combining with the benchmark portfolio R, it means that the latter is not efficient relative to r. Because the manager can always use the alternative of investing in the benchmark, a better portfolio than the benchmark portfolio can be formed, and the fund manager has added a source of value.

A graphical representation looks like this:

Figure 1



The figure represents two portfolios, A and B, of returns r_A and r_B, against a common benchmark. The "natural" (free of tax) portfolios are represented by $x_A(0)$ and $x_B(0)$; the "natural" positions are: long in fund A and short in fund B. For the fund A, the tax will be positive, because it derives value from going long, and for B, which derives value from going short, the tax will be negative.

Mathematically, we can determine *t*, the equivalent margin, by maximising the utility function with respect to x and solving the first order condition. *t* is the argument that maximises the expected utility function.

$$t_U = \arg\max E_{r,R} U[x(r-t)+(1-x)R]$$
$$EU[x(r-t)+(1-x)R] = EU[(r-R-t)x+R].$$
$$\frac{\partial EU}{\partial x} = E\{U'[(r-R-t)x+R]\cdot(r-R-t)\} = 0.$$

The equivalent margin is that value of *t* for which *x* is 0 (the tax that reduces the holding *x* to zero). That means:

$$E[(r-R-t)\cdot U'(R)]=0 \Leftrightarrow E[(r-R)\cdot U'(R)-t\cdot U'(R)]=0 \Rightarrow$$
$$\Rightarrow t_U=\frac{E[(r-R)\cdot U'(R)]}{E[U'(R)]}. \tag{1}$$

If we note $\pi(R)=\dfrac{U'(R)}{E[U'(R)]}$, it follows that $t_U=E[\pi(R)\cdot(r-R)]$. (2)

Thus, we can interpret the equivalent margin as representing the expected weighted sum of the differences between the fund return and the benchmark return, with weights *π(R)*. The weights are positive and sum in probability to unity. It means that the differences between the two returns are weighted with the marginal utilities of the benchmark realisations: states of the world in which the benchmark marginal utility is greater receive higher weightings. The weightings could be seen as a degree of risk-aversion. In this context, we could give an equivalent interpretation: states of the world in which the fund performs better than the benchmark receive a higher rate, depending on the investor's degree of risk aversion. So, when the fund performs well and the benchmark performs badly, the marginal utility *U'(R)* is high, and investors are interested in holding the fund.

Or, in terms of martingale measures, we could write expression (2) as $t_U=E^Q(r-R)$, where Q denotes a revised probability measure. In this world, investors are risk-neutral, and the welfare generated by *r* relative to *R* is nothing but the expectation of the difference *r-R*.

We can re-write expression (1) so as to introduce into the equivalent margin formula the regression of r on R.

Let us consider r as a function of R: $r=e(R)+\varepsilon$, where e(R) = E[r|R] and E[ε] = 0. e(R) represents the conditional expectation of r given R and ε is a disturbance term. Substituting this into (1) yields:

$$t_U=\frac{E[(e(R)-R)\cdot U'(R)]}{E[U'(R)]}. \tag{3}$$

From expression (3), it is obvious that t_U is positive if *e(R)>R* (because *U* is a Von Neumann – Morgenstern utility function: *U'*(▪)>0 and *U''*(▪)<0). This means that if for any given realisation of the benchmark R, the fund performs better, and the area between the theoretical regression curve of r on R and the 45º line is always positive.

This area could be regarded as a *cumulative investor surplus*, as suggested in Bowden (2001). Though, as a criterion for superiority it is too demanding to require *e(R)* to be greater than *R* for every *R*, because there may be temporary falls below the 45º line that does not have a serious impact on the overall welfare gain.

We could instead use the area between the regression curve of *r* on *R* and *R* itself, accumulated up to some given point *P*. This is what the OMD techniques, by large, does. If this area, pondered, as we shall see, with the relative frequency of *R*, is always positive, then we may conclude than the fund has been superior, over the period considered, to the benchmark.

It also obvious that *t* depends upon the particular utility function utilised to evaluate it. This inconvenient can be removed by introducing a *set of utility generators*, as named in Bowden (2000 and 2001), which have a strong connection with the second order stochastic dominance principle.

❑ 2.2. THE UTILITY FUNCTION AND THE POVERTY GAP FUNCTION. STOCHASTIC DOMINANCE

Merton (1981) shows that certain kinds of market timing can be regarded as equivalent to suitably chosen option strategies with puts or calls on the market index, using a non-linear theoretical regression of the fund return on the market: when the market return rises, the return on the fund rises more, and when the market index falls, the return on the fund falls less than the index. He showed that "the pattern of returns from successful market timing has an isomorphic correspondence to the pattern of returns from following certain option investment strategies where the (*implicit*) prices paid for options are less than their *fair* or market value."

He showed that the random variable end of period value of the assets, from the investor's point of view, can be written as:

$$V(t+1) = A(t) \cdot Z_M(t) + A(t) \cdot \max\left[0, R(t) - Z_M(t)\right], \text{ where:}$$

V(t+1) – the dollar return on the assets of the fund;

A(t) – the total value of investment in securities by the fund at time *t*;

$Z_M(t)$ – the return per dollar on the market;

R(t) – the return per dollar on the riskless asset.

According to this formula, the dollar return on a the assets of the fund is identical to the dollar return on a portfolio which follows the investment strategy of going long *A(t)* shares at $1 per share of the market portfolio and a put option on *A(t)* shares of the market portfolio with an exercise price per share of *R(t)*. This is a protective put strategy. It means that the successful market timer that follows the investment strategy described above will earn the gains, but is insured against the losses. If he had held only the market shares, he would have been exposed both to the gains and losses. The principal benefit of market timing, in this vision, seems to be providing insurance. In fact, the put option can be seen analogous to a term insurance policy, where the item insured is the value of the underlying asset and the face value of the policy (also called maximum coverage) - the exercise price.

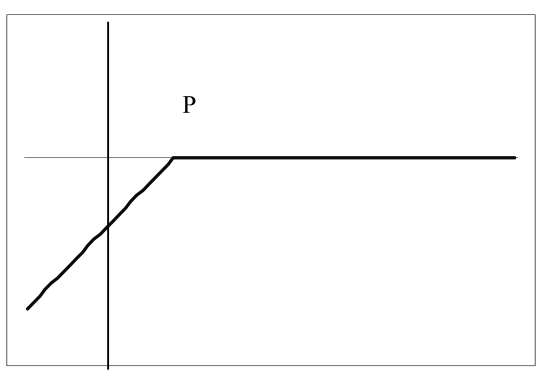
These reasons led Bowden (2000) to considering the utility function as having a short put pay-off profile with strike price P and the premium disregarded. For a fixed number P, the utility function is defined as:

$$U_P(y) = -\max(0, P - y), \text{ with } -\infty < y < \infty .$$

By a mathematical abuse, we consider that U'(P) exists; then $U_P'(y) \geq 0$, and $U_P''(y) < 0$, so that it is a von Neumann – Morgenstern utility function.

Graphically:

Figure 2



$$U_P(R) = -\max(0, P-R) = \begin{cases} R-P, R \le P \\ 0, R > P \end{cases}.$$

This type of utility function is motivated by the presumption that the investor is interested in obtaining a target return P, being indifferent to values of R in excess of this and negatively exposed if the return falls linearly below the target. The investor will fix his target according to his appetite for risk. A more complex interpretation of *P* will be made later in this paper.

The profile of the utility function comes very close to the so-called *poverty gap function*, as presented in Davidson and Duclos (2000), which is obtained on the stochastic dominance ground.

Considering two distributions of income[1] with cumulative distribution functions F_A and F_B, defined on R_+, we note:

$$D_A^1(x) = F_A(x) = \int_0^x dF_A(y) = \int_0^x f_A(y)dy,$$

Analogously we define:

$$D_B^1(x) = F_B(x) = \int_0^x dF_B(y) = \int_0^x f_B(y)dy.$$

where $f_A(y)$ and $f_B(y)$ represent the associated probability distribution functions.

We say that *A first order stochastically dominates B* (FSD) up to a poverty line *z* if $D_A^1 \le D_B^1$ (that is $F_A(x) \le F_B(x)$) for all $x \le z$ and $x \in R_+$. The graph of $F_A(x)$ will always be to the right of the graph of $F_B(x)$ as long as the FSD condition holds. The cumulative probability under the f_A distribution is less than (or equal to) the cumulative probability under the f_B distribution. The interpretation in terms of welfare economics is that "the headcount of individuals below the poverty line is always greater in B than in A for any poverty line not exceeding *z*" (Davidson-Duclos, 2000). Or, in other words, the probability of being beyond a given poverty threshold is greater in B (the dominated distribution) than in A (the dominant distribution).

[1] The term *income* is used by Davidson and Duclos in a larger sense, meaning a measure of individual welfare, not necessarily only money income.

The first order stochastic dominance holds only if the two distributions do not cross within the range of poverty lines. Otherwise, second order stochastic dominance (SSD) should be checked.

A second order stochastically dominates B (SSD) up to a poverty line *z* if $D_A^2 \leq D_B^2$, that is, $\int_0^x D_A^1(y)dy \leq \int_0^x D_B^1(y)dy$ for all x ≤ z.

The graphical explanation of SSD is: alternative A dominates alternative B if the area under the cumulative distribution function of A (F_A) is less than (or equal to) the area under the cumulative distribution function of B (F_B), or, equivalently, the cumulative area between F_B and F_A is non-negative for all x.

The derivation of the SSD condition is treated separately in what follows. For now, it is worth saying that, if FSD requires only that the decision maker's utility function should be increasing [$U'(\cdot)>0$], independent of concavity, the SSD rule requires a demising marginal utility of the income [$U''(\cdot) \leq 0$].

We must start by assuming that individuals are utility maximisers and their utility functions are of classical von Neumann – Morgenstern type, as mentioned above. Given two mutually exclusive alternative investments, A and B, with the probability functions f_A and f_B respectively, and cumulative distribution functions F_A and F_B respectively, we write that alternative A is preferred to B if:

$$\int_{-\infty}^{\infty} U(x) f_A(x)dx - \int_{-\infty}^{\infty} U(x) f_B(x)dx > 0 \text{, or}$$

$$\boldsymbol{I} = \int_{-\infty}^{\infty} U(x)\left[f_A(x) - f_B(x)\right]dx > 0 \text{, where } U(x) \text{ is the utility function.}$$

We integrate $\boldsymbol{I}$ by parts and obtain:

$$\boldsymbol{I} = \int_{-\infty}^{\infty} U(x)\left[F_A'(x) - F_B'(x)\right]dx =$$

$$= U(x)\left[F_A(x) - F_B(x)\right]_{-\infty}^{\infty} - \int_{-\infty}^{\infty} U'(x)\left[F_A(x) - F_B(x)\right]dx$$

As $\lim_{x \to \pm\infty} F(x) = 0$, it follows that:

$$\boldsymbol{I} = -\int_{-\infty}^{\infty} U'(x)\left[F_A(x) - F_B(x)\right]dx > 0 .$$

But $U'(x)>0$, from which it follows the definition of the first order stochastic dominance: A dominates B by FSD (***I*** >0) if $F_A(x) - F_B(x) < 0$.

We integrate once more ***I*** by parts:

$$\boldsymbol{I} = -\int_{-\infty}^{\infty} U'(x)\left[D_A'(x) - D_B'(x)\right]dx \text{ , where } D(x) = \int_{-\infty}^{x} F(y)dy$$

$$\boldsymbol{I} = \underbrace{\underbrace{-U'(x)}_{<0}\underbrace{\left[D_A'(x) - D_B'(x)\right]}_{<0}}_{>0} + \int_{-\infty}^{\infty} U''(x)\left[D_A(x) - D_B(x)\right]dx > 0$$

As *U''(x)* is negative, it follows that, in order to respect the presumption of A dominating B (***I*** >0), $D_A(x)–D_B(x)$ must be also negative. Thus we obtain the definition for second order stochastic dominance.

Generally, $D^s(x) = \int_0^x D^{s-1}dy$ for any $s \geq 2$, $s \in Z$.

Davidson and Duclos demonstrate that $D^s(x) = \frac{1}{(s-1)!} \cdot \int_0^x (x-y)^{s-1} dF(y)$.

Then, the SSD condition can be written as:

$$A \overset{SSD}{\succ} B \Leftrightarrow \int_0^x (x-y)dF_A(y) \leq \int_0^x (x-y)dF_B(y) \text{ for all } x \leq z \text{ .} \quad (4)$$

When the poverty gap is *z*, the *poverty gap function* is defined as:

$$g(z,y) = \max(z-y,0) = z - \min(y,z). \quad (5)$$

The last term – min(y,z) – is called *the censored income* defined for the poverty line z, and the difference *z-y* is called *the poverty gap*, denoting how far is the individual's income from the given poverty threshold *z*.

Using (5), it appears that $D^2(x)$ represents *the average poverty gap* up to *z*.

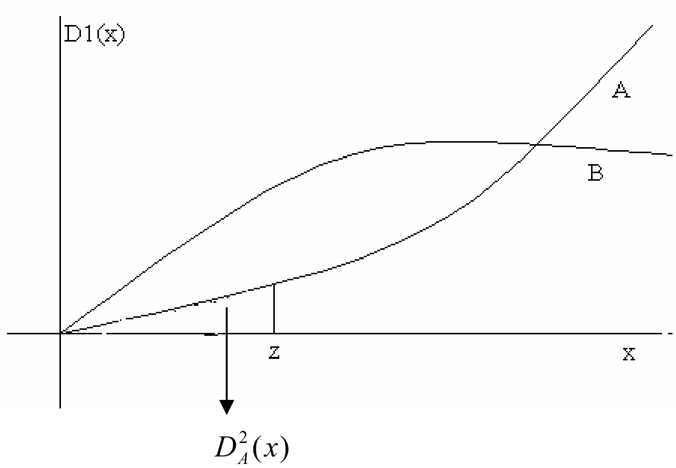
From (4) and (5) it follows that second order stochastic dominance up to *z* implies that the average poverty gap in B (the dominated distribution) – D_B^2 – is greater than in A (the dominant distribution) – D_A^2 – for all poverty lines less than or equal to *z*. There is a longer way from the actual level of income B to the poverty threshold than from the actual level of income A to the same poverty threshold.

Sometimes in literature[2] the graph of $D^1(x)$ is called *the poverty incidence curve*, and the graph of $D^2(x)$ – *the poverty deficit curve*.

The symmetry between the poverty gap function and the option-like utility function is clear: the latter is just the opposite of the former. We can interpret the poverty gap as a dis-utility function. Alternatively, we can see the utility function as showing, just like the poverty gap function, the difference between the actual level of income and the poverty threshold, but now the difference is positive: how far are we from the poverty threshold, *after* we surpassed the threshold.

Graphically, the first and the second order stochastic dominance is represented in figure 3, where alternative A dominates alternative B by SSD. The two distributions cross each other once.

Figure 3



One can find an interesting link to SSD in Rotschild and Stiglitz (1970), who show that the SSD condition can be thought as a preference for mean preserving inequality reducing changes in the distribution function. They consider two random variables, X and Y, with the corresponding density functions, *f* and *g*, respectively, where *g* is obtained by adding a so-called step function, *s*, to *f* so that *g* has the same mean as *f*, and adding *s* to *f* shits probability weight from the centre to the tails. If

[2] For example, Ravaillon, M (1994): *Poverty Comparisons, Fundamentals of Pure and Applied Economics*, Harwood Academic Publishers, Switzerland, quoted in Davidson and Duclos (2000)

$T(y) = \int_0^y [G(x) - F(x)]dx$, then $T(1) = 0$ and $T(y) \geq 0$. These are *the integral conditions*. T(y) would be, in terms of stochastic dominance theory, $D^2(y)$.

Setting that "a definition of greater uncertainty is a definition of partial ordering on a set of distribution functions," they define that $F \leq_I G$ (*F* is less risky than *G*, with riskiness judged as before: a variable is riskier than another having the same mean if it has more probability weight on the tails and less on the centre than the other) if and only if *G* - *F* satisfies the integral conditions, where *F* and *G* are the cumulative distribution functions of X and Y.

They further demonstrate that this approach to defining riskiness is equivalent to the more common definition involving von Neumann – Morgenstern utility functions (every risk averter prefers X to Y if $EU(X) \geq EU(Y)$). The partial ordering corresponding to this approach to riskiness is defined by Rotschild and Stiglitz as follows: "$F \leq_u G$ if and only if for every bounded concave utility function *U*, $\int_0^1 U(x)dF(x) \geq \int_0^1 U(x)dG(x)$," or $\int_0^1 U(x)[F(x) - G(x)]dx$.

X dominates Y if the integral is greater than 0. But since *U(x)* is a von Neumann – Morgenstern utility function, *U'(x)*>0. It follows that $\int_0^1 [F(x) - G(x)]dx \geq 0$, which is exactly the second order stochastic dominance condition.

2.3. The Theoretical Model for OMD

In section 2.1. was derived formula (3) for the equivalent margin:

$$t(P) = \frac{E[(e(R) - R) \cdot U'(R)]}{E[U'(R)]}, \qquad (3)$$

and, in the last section, we established the formula for the utility function, as in Merton (1981):

$$U_P(R) = -\max(0, P - R) = \begin{cases} R - P, R \le P \\ 0, R > P \end{cases}, \text{ and}$$

$$U'(R) = \begin{cases} 1, R \le P \\ 0, R > P \end{cases}.$$

We now develop the equivalent margin formula in order to obtain the ordered mean difference formula.

In what follows, *F(R)* and *G(r)* denotes the cumulative distribution functions for R and r, respectively, and *f(R)* and *g(R)* – the distribution functions for the two returns.

$$E[(e(R) - R) \cdot U'(R)] = \int_{-\infty}^{P} [e(R) - R] f(R) dR = \int_{-\infty}^{P} [e(R) - R] dF(R). \quad (6)$$

$$EU'(R) = \int_{-\infty}^{\infty} U'(R) f(R) dR = \int_{-\infty}^{P} \underbrace{U'(R)}_{1} \cdot \underbrace{f(R)dR}_{dF(R)} + \underbrace{\int_{P}^{\infty} \underbrace{U'(R)}_{0} \cdot f(R) dR}_{0}.$$

(Anyhow, we are not interested but up to the given point P.)

$$\Rightarrow EU'(R) = \int_{-\infty}^{P} dF(R) = F(R)\Big|_{-\infty}^{P} = F(P) - \underbrace{F(-\infty)}_{0} = F(P).$$

$$EU'(R) = F(P). \quad (7)$$

Introducing relations (6) and (7) into (3), we obtain:

$$t(P) = \frac{1}{F(P)} \int_{-\infty}^{P} [e(R) - R] dF(R) \text{ or}$$

$$t(P) = \frac{1}{F(P)} \int_{-\infty}^{P} [e(R) - R] f(R) dR \quad (8)$$

This is the schedule for the OMD. The result of this formula is called *the conditional ordered mean difference* (COMD) and represents the expected value of the ordered mean difference function at any point P. We will show that if COMD is positive over the entire range of R values, then the fund will be preferred to the benchmark by any risk averse investor, independently of his specific utility function. Thus, if *t(P)* ≥ 0 for all *P*, it will be concluded that the fund is superior to the benchmark even if temporarily *e(R)* happens to fall below *R*.

Graphically, expression (8) represents the average area between the regression line of *r* on *R* and *R* itself, accumulated up to the chosen point *P*, weighted with the relative frequency of the benchmark (*f(R)*), and divided by the number of observations (*F(P)* - the cumulative probability up to *P*, since $F(P)=\int_{-\infty}^{P} f(R)dR$)). The weights denote the degree of risk aversion.

An extension is to check for the degree of aggression or defensiveness of the fund relative to the benchmark. This is equivalent to check for the sign of *t'(P)*.

As stated before, the conditional ordered mean difference is nothing but the running mean of the differences between the return on the fund and the benchmark return. We can therefore assume the notation $E_P[\phi(R)]=\frac{1}{F(P)}\int_{-\infty}^{P}\phi(R)f(R)dR$ for *t(P)*, where $E_P[\bullet]$ denotes the running mean operator and $\phi(R)=e(R)-R$, the excess function.

Differentiating with respect to *P* gives:

$$\frac{dE_P\phi(R)}{dP}=-\frac{F'(P)}{F^2(P)}\cdot\int_{-\infty}^{P}\phi(R)f(R)dR+\frac{1}{F(P)}\phi(P)f(P)\,.$$

But *F'(P) = f(P)*; then:

$$\frac{dE_P\phi(R)}{dP}=\frac{f(P)}{F(P)}\cdot\left[\phi(P)-\underbrace{\frac{1}{F(P)}\cdot\int_{-\infty}^{P}\phi(R)f(R)dR}_{E_P\phi(R)}\right]\Rightarrow$$

$$\Rightarrow\frac{dE_P\phi(R)}{dP}=\frac{f(P)}{F(P)}\cdot\left[\phi(P)-E_P\phi(R)\right]. \qquad (9)$$

From equality (9), we can define the fund of return *r* as aggressive with respect to the benchmark (of return *R*) if for all values *R*, $t'(P)\geq 0$ (meaning that the OMD is increasing), and defensive if $t'(P)\leq 0$ (meaning that OMD is decreasing).

We showed until now that the theoretical OMD is the equivalent margin for a special sort of utility function, namely the put payoff–like function, in which utility is linear for *R*=*P* and 0 thereafter. The result can be generalised for any risk-averse utility function. Bowden (2000) demonstrates a very important result: he shows that the equivalent margin t_U (equality (3)) can be expressed as a weighted average of OMD schedules *t(P)* (equality (8)) for an arbitrary concave twice differentiable utility function *U*.

From (3):

$$t_U \cdot EU'(R) = E\left[\left(e(R) - R\right)U'(R)\right] = \int_{-\infty}^{\infty} \left[e(R) - R\right] \cdot U'(R) dF(R) . \tag{10}$$

From (8):

$$t(P) \cdot F(P) = \int_{-\infty}^{P} \left[e(R) - R\right] dF(R) . \tag{11}$$

Deriving (11) in point *R*, it follows that:

$$d\left[t(R) \cdot F(R)\right] = \left[e(R) - R\right] dF(R) . \tag{12}$$

Introducing (12) in (10), it results that:

$$t_U EU'(R) = \int_{-\infty}^{\infty} U'(R) \cdot d[t(R)F(R)] .$$

Integrating by parts (13), we obtain:

$$t_U EU'(R) = U'(R)t(R)F(R)\Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} U''(R)t(R)F(R)dR$$

$$t_U EU'(R) = U'(\infty)t(\infty)\underbrace{F(\infty)}_{1} - \underbrace{U'(-\infty)t(-\infty)\underbrace{F(-\infty)}_{0}}_{0} - \int_{-\infty}^{\infty} U''(R)t(R)F(R)dR$$

$$t_U EU'(R) = U'(\infty)t(\infty) - \int_{-\infty}^{\infty} U''(R)t(R)F(R)dR .$$

We divide both sides of the equation by *EU'(R)*:

$$t_U = \frac{U'(\infty)t(\infty)}{EU'(R)} - \frac{1}{EU'(R)} \int_{-\infty}^{\infty} U''(R)t(R)F(R)dR . \tag{13}$$

U being a concave utility function, we are entitled to consider that $U'(\infty) = 0$. On the other hand, if we note: $w(R) = -\frac{U''(R)F(R)}{EU'(R)}$, with $EU'(R) = \int_{-\infty}^{\infty} U'(R) f(R) dR$, then:

$$t_U = \int_{-\infty}^{\infty} w(R) t(R) dR ,$$

which is equivalent, if we change the variable of integration, with:

$$t_U = \int_{-\infty}^{\infty} w(P) t(P) dP , \quad (14)$$

where $w(P) = -\frac{U''(P)F(P)}{EU'(P)}$. (15)

Thus, we demonstrated that the equivalent margin is a weighted average of a set of OMD functions.

From (15), since *U''(P)* < 0, and *F(P)* and *EU'(P)* > 0, it follows that *w(P)* > 0.

We further show that the weightings *w(P)* sum in probability to 1. Returning to *R* as a variable of integration, we have:

$$\int_{-\infty}^{\infty} w(R) dR = -\int_{-\infty}^{\infty} \frac{U''(R)F(R)}{EU'(R)} dR =$$

$$= -\frac{1}{EU'(R)} \left[U'(R)F(R) \Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} U'(R) \underbrace{F'(R)}_{f(R)} d(R) \right] =$$

$$= -\frac{1}{EU'(R)} \left[\underbrace{U'(\infty)}_{0} \underbrace{F(\infty)}_{1} - U'(-\infty) \underbrace{F(-\infty)}_{0} - \underbrace{\int_{-\infty}^{\infty} U'(R) f(R) dR}_{EU'(R)} \right] =$$

= 1.

So, $\int_{-\infty}^{\infty} w(P) dP = 1$.

Thus, we can write the equivalent margin t_U as:

$$t_U = \int_{-\infty}^{\infty} w(P) \cdot t(P) dP \text{, where}$$

$$w(P) = -\frac{1}{2} \cdot \frac{U''(P)}{EU'(R)} \cdot F(P), \qquad w(P) \geq 0; \int_{-\infty}^{\infty} w(P) dP = 1.$$

w(P) represents weights denoting the degree of risk aversion of the investor.

From expression (14) it follows that if $t(P) \geq 0, \forall P$, then the equivalent margin t_U will also be positive, meaning that the fund performed better than the benchmark.

It also can be seen from (14) that the equivalent margin can be decomposed in two factors:

- the effect due to the fund return: *t(P)*
- the effect due to the investor's risk degree of risk aversion: *w(P)*

The immediate idea following from the fact that the equivalent margin is a weighted average of OMD schedules is that the investor can be seen as a spectrum of elementary investors ("gnomes", as Bowden (2001) names them), each of them having a put option profile utility function and the "gnomes" differing from each other by the "strike price", or, "focal point" (*P*). Each gnome has a different degree of risk aversion. So, every investor is viewed as having attached different weights at every level of risk aversion, denoted by a focal point. The risk aversion decreases as the focal point *P* moves to the right (but in inter-personal comparisons, for the same level *P*, the degree of risk aversion may have different values) because the investor fixes a higher level of performance for the fund, so he is willing to accept a higher level of risk. As *P* moves to the right, the risk premium declines, and this can relatively easy be deduced from the formula of *w(P)*.

The spectrum of weights attached to the focal points for every "gnome" is what makes the difference between investors concerning their appetite for risk: an investor who has a higher proportion of "gnomes" with lower focal points is more risk averse than another investor who has a higher proportion of "gnomes" with higher focal points.

2.4. Testing for Stochastic Dominance via OMD

The definition of stochastic dominance was given in the preceding paragraph. In the present paragraph we will show it is not necessary to drive a separate test for stochastic dominance as long as we test for OMD because the OMD criterion is a sufficient condition for stochastic dominance.

According to the definition presented earlier in this paper, we say that r dominates R by SSD if and only if $\int_{-\infty}^{P} G(r)dr \le \int_{-\infty}^{P} F(R)dR$, for all P, where $G(r)$ and $F(R)$ denote the cumulative distribution functions for r and R, respectively.

$$r \overset{SSD}{\succ} R \Leftrightarrow \int_{-\infty}^{P} G(r)dr \le \int_{-\infty}^{P} F(R)dR. \qquad (15)$$

One can infer:

$$\begin{aligned} EU_P(R) &= \int_{-\infty}^{\infty} U(R)f(R)dR = \int_{-\infty}^{P} U(R)f(R)dR + \int_{P}^{\infty} U(R)f(R)dR = \\ &= \int_{-\infty}^{P} [-(P-R)]f(R)dR + \underbrace{\int_{P}^{\infty} 0 \cdot f(R)dR}_{0} = \\ &= -\int_{-\infty}^{P} (P-R)f(R)dR = \\ &= -\int_{-\infty}^{P} (P-R)dF(R) = \\ &= -\int_{-\infty}^{P} F(R)dR. \end{aligned}$$

In a similar way:

$$EU_P(r) = -\int_{-\infty}^{P} G(r)dr.$$

It follows that expression (15) is equivalent with this one:

$$r \overset{SSD}{\succ} R \Leftrightarrow EU_P(r) \ge EU_P(R), \text{ for any concave } U. \qquad (16)$$

This finding comes to hold the result of Rotschild and Stiglitz (1970), who showed that defining risk by means of how much probability weight a variable has in the tails and its centre is similar to the definition of risk invoked by the theory of expected utility maximisation. The discussion has been carried out in paragraph 4.

In words, if r dominates R by SSD, then $t(P) \geq 0$, $\forall P$; if there exists a P such that $t(P) < 0$, then r does not dominate R by second order stochastic dominance. We can therefore create a sufficiency test for SSD using the OMD formula. To demonstrate that, we have to take r as a benchmark and evaluate R against r.

All across the present paragraph, in order to avoid confusion, we will use the following notations: $t_R(P)$ will be the OMD function for R, with r as benchmark, and $t_r(P)$ will be the usual OMD function, as defined in the preceding paragraph, with R as benchmark for r. To define $t_R(P)$, we have to introduce, as before for r, the regression function of R on r: $E[R \mid r] = \gamma(r)$. The analogous regression, of r on R, has been defined before as $E[r \mid R] = e(R)$. We then have the two OMD functions:

$$t_R(P) = \frac{E_r\left[\left(e(R) - R\right) \cdot U'_P(R)\right]}{EU'_P(R)} \quad \text{and}$$

$$t_r(P) = \frac{E_r\left[\left(\gamma(r) - r\right) \cdot U'_P(r)\right]}{EU'_P(r)}.$$

Demonstrating that r dominates R by SSD reduces to demonstrating that COMD with r as benchmark is semi-negative for every P. Or, mathematically written:

$$t_r(P) \leq 0, \forall P \Rightarrow r \overset{SSD}{\succ} R.$$

We start from the inequality:

$$U(r) \leq U(R) + (r - R) \cdot U'(R)$$

Applying the mean operator over the joint distribution, it follows that:

$$E_r U(r) \le E_R U(R) + E_R\left[\left(E_r[r \mid R] - R\right)\cdot U'(R)\right] =$$

$$= E_R U(R) + E_R\left[\left(e(R) - R\right)\cdot U'(R)\right] =$$

$$= E_R U(R) + t_R(P)\cdot EU'(R).$$

Similarly,

$U(R) \le U(r) + (R - r)\cdot U'(r)$ and

$$E_R U(R) \le E_r U(r) + E_r\left[\left(E_r[R \mid r] - r\right)\cdot U'(r)\right] =$$

$$= E_r U(r) + E_r\left[\left(\gamma(r) - r\right)\cdot U'(r)\right] =$$

$$= E_r U(r) + t_r(P)\cdot EU'(r).$$

(All utility functions considered in this demonstrations are the utility functions of the "gnomes", so we had, to be more correct, to write them with the subscript $_P$.)

We have:

$$\begin{cases} E_r U(r) \le E_R U(R) + t_R(P)\cdot EU'(R) \\ E_R U(R) \le E_r U(r) + t_r(P)\cdot EU'(r). \end{cases} \qquad (17) \text{ and } (18)$$

From inequality (18):

$$-t_r(P)\cdot EU'(r) \le E_r U(r) - E_R U(R) \qquad (19)$$

and from inequality (17) we have:

$$E_R U(R) - E_r U(r) \le t_R(P)\cdot EU'(R) \qquad (20)$$

Put together, inequalities (19) and (20) give:

$$-t_r(P)\cdot EU'(r) \le E_r U(r) - E_R U(R) \le t_R(P)\cdot EU'(R). \qquad (21)$$

It is obvious in inequality (21) that if $t_r(P) \le 0$, then, as $U'(r)>0$, $E_rU(r) \ge E_RU(R)$, and r dominates R by SSD. In other words, we showed that if $t_R(P)<0$ for all P, then r second order stochastically dominates R.

Concluding, we can establish the following:

– if r dominates R by SSD, then $t_R(P) \geq 0$;
– if $t_r(P) \leq 0$, for all P, then r dominates R by SSD.

In other words, $t_R(P) \geq 0$ is a necessary, but not sufficient condition for r to SSD R. SSD rule needs also that $t_r(P) \leq 0$.

It may happen a fund to be OMD dominant over the benchmark, but dominated by the market in terms of SSD.

However, the OMD criterion is superior to SSD by the fact that, while the SSD condition is cast in terms of marginal distributions of the two series of returns, the OMD involves joint distribution of returns r and R. The SSD criterion ignores the conditionally by retaining only the marginal returns of two funds, so it does not utilise important information about market timing.

2.5. THE ECONOMETRIC MODEL

Either a parametric or a non-parametric approach can be employed to estimate the COMD. The non-parametric estimate will be treated later in this paper and we will use it to verify the result obtained by parametric testing. The non-parametric method, though it has the advantage of simplicity, has some inconvenience that makes it improper.

We have to estimate the function in expression (8), which is the OMD function and its value at point P is the COMD.

$$t(P) = \frac{1}{F(P)} \int_{-\infty}^{P} [e(R) - R] f(R) dR \qquad (8)$$

Two series of return are needed:

- the observations for the benchmark, that will be noted as R_1, R_2, …, R_N, and will represent the exogenous variables;
- the observations for the fund return, that will form the vector of the dependent variables: $r_i,\ i = \overline{1, N}$

We proceed by specifying a functional form for the theoretical regression of the fund return, *r*, on the benchmark return, *R*. We note it *e(R, θ)*. By using a transformation that will immediately be presented here, we will make the regression as a function of a parameter *G* instead of *R*, so as the regression polynomial be an orthogonal polynomial, which has some desirable properties. *G* is obtained from *R* by means of Forsyth polynomials.

The initial regression of *r* on *R* is: $r_i = e(R_i, \theta) + \varepsilon_i$, where ε_i are random variable with zero mean and constant variance. At this point, a transformation will be made. It is desirable that the independent terms of the regression polynomial should be orthogonal, so that the variables R_i not to be correlated. The inexistence of correlation between the independent variables in a regression model implies that the multiple regression slopes are equal to the slopes in the individual simple regressions. This is an important property for our model because it will be constructing by eliminating some terms. In order to insure that the regression polynomial is orthogonal, we will use the Forsyth polynomials, that transform the initial polynomial in an orthogonal one.

Using the Forsyth polynomials, we construct a set G_0, G_1, G_2,…G_K of new regressors, with order *K* chosen at will, as follows: for each observation *i = 1,2,…N*, we define:

$$g_{i,k} = (R_i - \varphi_k) \cdot g_{i,k-1} - \rho_k \cdot g_{i,k-2}, \text{ where}$$

$$\varphi_k = \frac{\sum_{i=1}^{N} R_i \cdot g_{i,k-1}^2}{\sum_{i=1}^{N} g_{i,k-1}^2} \text{ and } \rho_k = \frac{\sum_{i=1}^{N} R_i \cdot g_{i,k-1} \cdot g_{i,k-2}}{\sum_{i=1}^{N} g_{i,k-2}^2}, \text{ for } k = 1, 2, \ldots K.$$

We need to specify $g_{i,-1}$= 0 (not used among the regressors) and $g_{i,0}$= 1, and consider $g_{i,-1}^2 = 0$. Proceeding this way, we obtain an orthogonal matrix G'G (all the off diagonal elements are zero), where G is the matrix of the regressors.

The new regression model is defines as a function of G_i and parameters *θ*. (From now, we will use capital letters for G):

$$r_i = e(G_i, \theta) + \varepsilon_i \text{, where } e(G_i, \theta) = \sum_{k=0}^{K} \theta_k \cdot G_{i,k} .$$

The regression polynomial that will be used to estimate the OMD (by OLS) will be:

$$r_i = \theta_0 + \theta_1 \cdot G_{i,1} + \theta_2 \cdot G_{i,2} + ... + \theta_K \cdot G_{i,K} \tag{22}$$

Until now we will have obtained just an estimation for *r*. Our objective is to estimate *t(P)*, not *r*. After applying OLS (ordinary least squares method), we obtain the estimates of the parameters, $\hat{\theta}$, and the fitted values of *r*, $\hat{e}_i = e(G_i, \hat{\theta})$. The next step is to work out the values of t_j at each sample point j = 1, 2,…N, obtaining a set of estimated values of *t(P)*:

$$\hat{t}_j = \frac{1}{j} \sum_{i=1}^{j} (\hat{e}_i - R_i) \text{, where } \hat{e}_i = e(G_i, \hat{\theta}) . \tag{23}$$

3. EMPIRICAL APPLICATION

❑ 3.1. THE DATA

In this section we will apply the econometric model described in the preceding section on a set of real data. The two series of data are represented by:

- the monthly return of a mutual fund, respectively Capital Plus, managed by Certinvest management company, calculated as the change in the net asset value (NAV) per share;
- the monthly rising in the mutual fund index (MFI), as the benchmark.

Capital Plus is one of the first Romanian mutual funds. It was funded in 1995 and started its activity on September 6, 1995, under the initial name of Credit Fond; beginning with July 20, 1999, it changed its name in Capital Plus. The management company is Certinvest, which exists from September 1994, and has another two mutual funds under administration: Tezaur, a money market fund, established in June 30, 1999, and Intercapital. The custodian is the Romanian Bank for Development - Société Générale.

According to the Prospectus, the investment policy of Capital Plus is to obtain the highest possible return while maintaining all the liquidity and risk parameters at the levels required by the National Commission of Securities[3] (CNVM). Capital Plus is situated in the middle risk class (the second from four in ascending order of risk), according to the methodology presented in Settlement no. 9, which is the fundamental act in the Romanian mutual fund legislation.

The majority of the Romanian mutual funds are money market funds because of the poor development of the exchange market, and very few mutual funds invest in stocks and corporate bonds. Capital Plus is one of them.

The **mutual fund index** (MFI) is a unofficial index of the Romanian mutual funds, which is calculated since July 10, 1998, and was officially launched on September 18, 1998. The index is constructed after the model of the exchange indexes. It is calculated weekly on the basis of the net asset value and the net asset value per share of the selected mutual funds. Presently, there are 20 mutual funds which enter the portfolio of the mutual fund index. The reference data is July 10, 1998, when to the index was attributed the value 1,000 points. The calculus formula of the index is:

$$IFM_k = \sum_{i=1}^{N} \left(\frac{C_k^i}{C_0^i} \cdot \frac{A_0^i}{A_0^T} \right) \cdot 1000 \text{, where:}$$

- IFM_k: the value of the index in the day before the day for which it is calculated (k);
- N: the number of funds that enter in the composition of the index;
- C^i_k: NAV per share of the fund i, on day k;
- C^i_0: NAV per share of the fund i, on the reference day;
- A^i_0: NAV of the fund i, on the reference day;
- A^T_0: NAV of the fund i, on the reference day.

The reason for choosing this index as a benchmark was that it seemed the most adequate. The BET index has initially been tested, but it proved unable to provide a satisfactory regression.

[3] the Romanian homologue for the Security and exchange Commission in the United States

Unlike the inspiring article (Bowden, 2000), who uses monthly data, I used weekly data, from January 3, 2000 to April 1, 2002, including 118 observations. The reason for this deviation from the original is the non-existence of sufficiently long time series, as the Romanian mutual funds are very young. A drawback arising from this is that the calculus with the Forsythe polynomials are more laborious, there are more explanatory variables in the regression model.

Table A.1. in the annex presents the two series of returns (in percents).

❑ 3.2. REGRESSION ESTIMATION

As shown in Section 2.5, the initial form of the regression polynomial just doesn't matter. Practically, one does not need to have a regression of the fund return on the benchmark. The two series of return alone are all we need for the beginning. Actually, a quadratic regression of the fund return on the benchmark return has plotted, but it has not exhibited any evidence that it was a correct presumption (the R^2 was 0.18). We are therefore obliged to try the Forsyth transformation.

We have the two time series and apply to them the Forsythe polynomial in order to obtain an orthogonal regression polynomial. Proceeding thus, we ensure of the absence of multicollinearity among the explanatory variables of the regression model, as shown in Section 2.5. There was also shown there that the regression polynomial will be constructed having as regressors a set G_0, G_1, …, G_K, which are the output of the Forsythe polynomial. In an Excel sheet I calculated the values for 33 such regressors. I chose the order K to be 33 to ensure of the completitude of the equation. 33 is the maximum number technically allowable by the spread sheet because of the high exponential order of the numbers representing intermediate calculus and values of regressors. Anyway, the econometric estimation demonstrated that 26 regressors would have been sufficient for a 5% confidence interval for the Student statistic.

Practically, applying the Forsyth transformation means that the mutual fund index (the benchmark) is divided into several virtual indices, independent of each other, and we shall have a regression of the fund return on those indexes instead of having a regression of the fund return on a single index.

After applying the Forsyth polynomial, I obtained a gross regression equation having the form:

$$VUAN_i = \theta_0 \cdot G_{i,0} + \theta_1 \cdot G_{i,1} + \theta_2 \cdot G_{i,2} + ... + \theta_{20} \cdot G_{i,33} + \varepsilon_i, \qquad (24)$$

where $VUAN_i$ is return on the fund, denoting the rising in net asset value per share, and, as specified before, $G_{i,0} = 1$ for all $i = 1, 2, \ldots 118$.

The estimation was made using the E-Views 3.0 econometric program. The output in table 1 (page 32)

The value of R^2 (0.76) indicates a good measure of the goodness of fit of the equation. The value of the Durbin Watson statistic (1.97) indicates that the (positive) correlation between residuals is insignificant. The matrix of regressors is orthogonal, as we expected after applying the Forsythe polynomial, meaning that there is no multicollinearity between explanatory variables.

The next step is to obtain the regression equation that will be used in computing the COMD. The procedure is to eliminate the terms in the gross equation using as a criterion the t-statistic with at a significance level of 5%. The far right column reported by E-Views (*Prob*) indicates that we should not reject the null hypothesis that the slope coefficient is 0 in the cases of the terms *G2*, *G4*, *G9*, *G10*, *G11*, *G12*, *G13*, *G16*, *G17*, *G18*, *G19*, *G20*, *G21*, *G23*, *G24*, *G27*, *G28*, *G29*, *G31*, *G32* and *G33*. This means that we will drop these regressors when constructing the new equation regression. The orthogonality of the matrix of regressors ensures us that the slopes of the remaining coefficients will be unchanged.

The new regression equation will be:

$$\begin{aligned} VUAN_i = {} & \theta_0 G_{i,0} + \theta_1 G_{i,1} + \theta_3 G_{i,3} + \theta_5 G_{i,5} + \theta_6 G_{i,6} + \theta_7 G_{i,7} + \theta_8 G_{i,8} + \theta_{14} G_{i,14} + \theta_{15} G_{i,15} + \\ & + \theta_{22} G_{i,22} + \theta_{25} G_{i,25} + \theta_{26} G_{i,26} + \theta_{30} G_{i,30} + \varepsilon_i . \end{aligned}$$

Table 1: OLS estimation of eqation (24)

Dependent Variable: VUAN
Method: Least Squares
Sample: 1/03/2000 4/01/2002
Included observations: 118

Variable	Coefficient	Std. Error	t-Statistic	Prob.
G0	0.844308	0.016420	51.41817	0.0000
G1	0.721442	0.089822	8.031870	0.0000
G2	0.231633	0.296339	0.781651	0.4366
G3	-3.706504	0.918718	-4.034432	0.0001
G4	-5.779245	3.108811	-1.858989	0.0665
G5	67.27390	13.13017	5.123611	0.0000
G6	-205.8248	51.42069	-4.002763	0.0001
G7	634.0910	180.6416	3.510216	0.0007
G8	-4115.715	592.9830	-6.940697	0.0000
G9	790.3135	2412.627	0.327574	0.7440
G10	2326.279	10287.52	0.226126	0.8217
G11	4759.479	52970.16	0.089852	0.9286
G12	268513.3	283858.2	0.945942	0.3469
G13	3412154.	1763473.	1.934905	0.0564
G14	25329806	9918047.	2.553911	0.0125
G15	2.80E+08	58030655	4.830550	0.0000
G16	6.89E+08	3.51E+08	1.964330	0.0528
G17	4.52E+09	2.28E+09	1.979642	0.0510
G18	1.55E+10	1.45E+10	1.071569	0.2870
G19	1.71E+11	9.02E+10	1.898840	0.0610
G20	5.40E+11	6.04E+11	0.894064	0.3738
G21	-7.06E+12	3.90E+12	-1.810508	0.0738
G22	-4.80E+13	2.33E+13	-2.059404	0.0426
G23	8.51E+13	1.28E+14	0.665235	0.5077
G24	-9.27E+14	8.82E+14	-1.050649	0.2964
G25	-1.24E+16	5.43E+15	-2.279886	0.0251
G26	-1.05E+17	3.41E+16	-3.092572	0.0027
G27	-2.04E+17	2.71E+17	-0.752069	0.4541
G28	-1.97E+18	2.09E+18	-0.942986	0.3484
G29	-1.11E+19	1.54E+19	-0.724615	0.4707
G30	-2.20E+20	1.10E+20	-1.999961	0.0487
G31	-8.47E+20	8.30E+20	-1.020105	0.3106
G32	3.15E+21	6.20E+21	0.508184	0.6127
G33	1.42E+22	3.97E+22	0.357082	0.7219

R-squared	0.760581	Mean dependent var	0.844309
Adjusted R-squared	0.666524	S.D. dependent var	0.308882
S.E. of regression	0.178371	Akaike info criterion	-0.373495
Sum squared resid	2.672571	Schwarz criterion	0.424837
Log likelihood	56.03623	F-statistic	8.086360
Durbin-Watson stat	1.970102	Prob(F-statistic)	0.000000

After re-applying the ordinary least squares method, we eliminate the terms *G22* and *G30* because of their higher than 0.05 p-values. The remaining equation, which will be used in determining the COMD, therefore, is:

$$VUAN_i = \theta_0 G_{i,0} + \theta_1 G_{i,1} + \theta_3 G_{i,3} + \theta_5 G_{i,5} + \theta_6 G_{i,6} + \theta_7 G_{i,7} + \theta_8 G_{i,8} + \theta_{14} G_{i,14} + $$
$$+ \theta_{15} G_{i,15} + \theta_{25} G_{i,25} + \theta_{26} G_{i,26} + \varepsilon_i \,. \quad (25)$$

The output produced by E-Views is presented in table 2:
(Another way to get this result could have been the stepwise regression procedure, as indicated in Bowden (2000), but the references in the econometric literature suggest that this method is rather inappropriate).

Table 2: OLS estimation of regression equation (25)

Dependent Variable: VUAN
Method: Least Squares
Sample: 1/03/2000 4/01/2002
Included observations: 118

Variable	Coefficient	Std. Error	t-Statistic	Prob.
G0	0.844309	0.017582	48.02112	0.0000
G1	0.721467	0.096177	7.501475	0.0000
G3	-3.706143	0.983709	-3.767520	0.0003
G5	67.27757	14.05902	4.785365	0.0000
G6	-205.8137	55.05829	-3.738106	0.0003
G7	634.1467	193.4205	3.278590	0.0014
G8	-4115.562	634.9318	-6.481895	0.0000
G14	25329839	10619671	2.385181	0.0188
G15	2.80E+08	62135869	4.511404	0.0000
G25	-1.24E+16	5.82E+15	-2.129257	0.0355
G26	-1.05E+17	3.65E+16	-2.888251	0.0047

R-squared	0.650351	Mean dependent var	0.844309
Adjusted R-squared	0.617674	S.D. dependent var	0.308882
S.E. of regression	0.190990	Akaike info criterion	-0.384610
Sum squared resid	3.903045	Schwarz criterion	-0.126326
Log likelihood	33.69198	F-statistic	19.90212
Durbin-Watson stat	2.036243	Prob(F-statistic)	0.000000

We start by verifying the OLS presumptions. Table 3 (page 34) synthesises the results presented above concerning the assumption of the OLS estimation.

- **Durbin Watson test**

As the **Durbin-Watson** statistic indicates (see tables 2 and 3), there is no evidence of first order correlation between residuals. The value of the statistic is 2.03, which is close enough to 2 to accept the null hypothesis of absence of first order autocorrelation. We check for higher degrees of autocorrelation by carrying other econometric tests.

Table 3 – Summary of statistics for the equation regression (25)

Test	Distribution	Value	5% critical value
Non-autocorrelation between residuals			
Durbin-Watson	–	2.036234	(2.00)
Breusch-Godfrey	χ^2 (4)	0.875133	9.49
(LM Test)	χ^2 (8)	9.214078	16.92
	χ^2 (16)	16.78303	26.30
	χ^2 (32)	30.48619	45.91
Ljung-Box	χ^2 (4)	0.7470	9.49
(Q-Statistic)	χ^2 (8)	7.5695	16.92
	χ^2 (16)	13.060	26.30
	χ^2 (32)	35.822	45.91
Homoscedasticity			
White	χ^2 (41)	19.91572	56.66
	R^2	0.168777	–
Koenkar-Basset	χ^2 (8)	0.032296	14.07
Engle (ARCH LM)	χ^2 (4)	0.052959	9.49
	χ^2 (8)	0.533095	15.51
	χ^2 (16)	1.212054	26.30
	χ^2 (32)	19.88728	45.91
Standard normality of residuals			
Skewness	–	0.488062	(0.00)
Kurtosis	–	4.332561	(3.00)
Jarque-Berra	χ^2 (2)	13.30163	5.99

- **Breusch-Godfrey Test**

The result of the LM test with 4, 8, 16 and 32 lags is presented in table 4 above. The tabulated value for the χ^2 statistic with 4 degrees of freedom at a 5% level of significance is much greater than the Breusch-Godfrey LM statistic for all the levels of significance. Therefore, the null hypothesis of no autocorrelation will be clearly accepted in all the cases. The very low values of the statistic indicate that probably the test would produce the same results even at 1% significance level.

- **Ljung-Box Q statistic and correlogram**

Table 3 shows that we cannot speak of autocorrelation up to 36th order in terms of Ljung-Box statistic neither. The Q values are less than the corresponding critical values for the χ^2 distribution (with degrees of freedom equal to the number of lags), which leads to the acceptance of the null hypothesis of no autocorrelation.

The autocorrelation (AC) and partial autocorrelation (PAC) figures fall between the two standard error bounds, meaning that they are not significantly different from zero at (approximately) the 5% significance level.

- **White Test**

The E-Views output for the White heteroscedasticity test is presented in table 4:

Table 4 – White Test

White Heteroscedasticity Test:			
F-statistic	0.376380	Probability	0.999528
Obs*R-squared	**19.91572**	Probability	0.997755

The White statistic (Obs*R-squared) is asymptotically distributed as a χ^2 distribution with degrees of freedom equal to the number of slope coefficients in the test regression, respectively 41[4]. The critical value for χ^2 with 41 degrees of freedom, at a 5% significance level, is 56.66, which is greater than the value of the White statistic for our model (19.915). We should therefore accept the null hypothesis of homoscedasticity. The high p-value for the F-statistic leads to the same decision.

More, the R^2 coefficient of the auxiliary regression (0.168777) is insignificant, also leading to accepting the null hypothesis of homoscedasticity. (I omit to present the table with the E-Views estimation output because it is too large, and from that table we are interested only in the value of the R^2 coefficient).

Actually, the White test is more general, in the sense that it refers not only to the heteroscedasticity aspect, but also to other aspects concerning the (possible) misspecification of the model, such as the residuals' independence of the regressors. Obtaining an insignificant test statistic implies that neither the homoscedasticity, nor the errors' independence of the regressors is violated.

- **Koenkar-Basset test**

I performed this test instead of Breusch-Pagan test because of the non-normality of the residuals (as it will later be shown), and, as Greene (1993) points out, "the Breusch-Pagan test is quite sensitive to the assumption of normality". Under the

[4] The test regression is a regression of the square of the residual term having as regressors the cross products of the explanatory variables in the initial regression model. In this particular case, we have 11 regressors in the regression model, so we'll have 41 regressors in the test regression, after excluding the redundant cross products

hypothesis of homoscedastivity, the Koenkar-Basset statistic is, as the Breusch-Pagan statistic, asymptotically distributed as χ^2 with degrees of freedom equal to the number of variables in the estimated function.

The Koenkar-Basset test also leads to the rejection of the alternative hypothesis of heteroscedasticity presumption: the statistic is 0.032296, while the critical χ^2 value is 14.07. The statistic was computed using as explanatory variables for the function that is estimated the regressors *G0*, *G1*, *G3*, *G4*, *G5*, *G6*, *G7* and *G8* of the regression equation. I could not use the entire set of regressors because their matrix is a singular one, so it could not be inverted, and the inverse matrix of regressors is required in the computation of the test statistic[5].

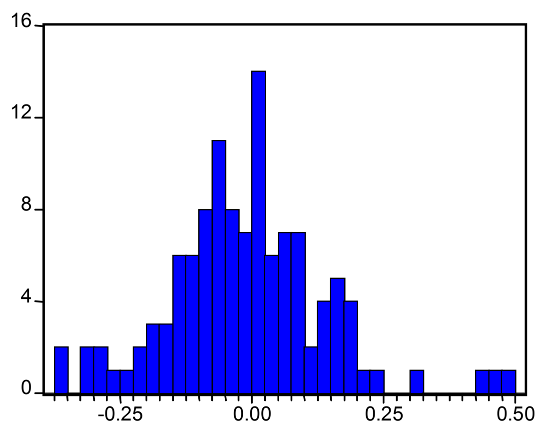
- **ARCH LM test (Engle)**

This is a test for determining whether autoregressive conditional heteroscedasticity effects are present in the residuals. Table 3 shows the statistics and the corresponding critical χ^2 values for several lags. In all cases we accept the null hypothesis that there are no ARCH effects up to that lag.

- **Jarque-Berras test**

E-Views produces the following output (figure 4):

Figure 4



Series: RESID	
Sample 1/03/2000 4/01/2002	
Observations 118	
Mean	-0.009717
Median	-0.013709
Maximum	0.485411
Minimum	-0.357526
Std. Dev.	0.149699
Skewness	0.488062
Kurtosis	4.332561
Jarque-Bera	13.30163
Probability	0.001293

The mean is insignificantly different from zero, but the hypothesis of normality is clearly rejected by the high value of the Jarque-Bera statistic and the significant values of skewness and kurtosis, which indicate that the residual series is

[5] The hypothesis of homoscedasticity is being tested against the alternative hypothesis that the variance has the form: $\sigma_i^2 = \sigma^2 \cdot f(\alpha_0 + \alpha_1 z_{i,1} + ... + \alpha_p z_p)$. If the model is homoscedastic, then α_l=0.

both asymmetric (skewness greater than 0) and leptokurtic (kurtosis greater than 3). From the histogram, one can observe that there are some extreme values in the residual series that may be the main cause in the non-normality. We could therefore check for normality after eliminating these extreme values. The result indeed suggests that without the extreme value the residual series appears to be closer to a standard normal distributed series with zero mean and constant variance: the new skewness is -0.205514 and the new kurtosis, 3.058781. It follows that the new Jarque-Bera statistic is 0.818897 (and the p-value 0.664016), which is closer to the critical value of 5.99 (a χ^2 distribution with 2 degrees of freedom, for a 5% confidence interval).

The problem of non-gaussian errors is treated in Hamilton (1994), pp. 208-214. It is demonstrated that under non-Gaussian conditions, the estimated parameters of the regression model continue to be unbiased and consistent, and their asymptotic distribution can be approximated by the asymptotic distribution of the estimated parameters under conditions of Gaussian disturbances. It is also shown there that the critical values for the t and F tests do not differ significantly if the sample is sufficiently large.

Testing the assumption of the independence between errors and regressors

A simple way of testing is to look at the covariance matrix. Table 5 below presents the covariances between the residuals and each regressor.

Table 5 - Covarriances between residuals and regressors

G	G0	G1	G3	G5	G6	G7
RESID	-1.04E-32	-3.24E-18	-8.61E-20	4.83E-20	2.15E-20	-1.49E-21

Table 5 (cont'd)

G	G8	G14	G15	G25	G26
RESID	-8.46E-22	-3.78E-26	-1.93E-26	6.22E-35	-4.15E-36

We cannot find any evidence of correlation between residuals and explanatory variables, which means that the independence presumption required by the OLS method is satisfied.

Although the dependent variable in the model (VUAN) is non-stationary (as well as the IFM series), the residuals series is stationary, which means that the Gauss-Markov theorem holds: the OLS estimators are the most efficient linear unbiased estimators. The ADF and Phillip-Perron tests have been used to check for the residuals' staionarity. The results are presented in table 6 and 7 (page 38):

Table 6 – Unit root ADF test for residuals series

ADF Test Statistic	-3.744255	1% Critical Value*	-3.4890
		5% Critical Value	-2.8870
		10% Critical Value	-2.5802
*MacKinnon critical values for rejection of hypothesis of a unit root.			

Table 7 – Unit root Phillips-Pérron test for residuals series

PP Test Statistic	-10.93526	1% Critical Value*	-3.4870
		5% Critical Value	-2.8861
		10% Critical Value	-2.5797
*MacKinnon critical values for rejection of hypothesis of a unit root.			
Lag truncation for Bartlett kernel: 4			(Newey-West suggests: 4)

Both the ADF and the Phillip-Pérron tests indicate the rejection of the hypothesis of existence of a unit root in the residuals series.

On the other hand, the fact that both VUAN and IFM are non-stationary, but the residuals of the regression of VUAN on IFM are stationary suggests that the two series must be cointegrated. This question is to be treated in Section 3.5.

❑ 3.3. Computation of OMD. Exponentially Weighted OMD

The next step is to compute the conditional ordered mean difference. We have therefore to estimate the function (8) at each point of the sample. Formula (8) represents, as noted at that point, the average area between the regression line of *r* on *R* and *R* itself, accumulated up to the chosen point *P*, weighted with the relative frequency of the benchmark, and divided by the number of observations, or the average difference between the values of the fund return and the benchmark return. It is therefore straightforward to think of an estimator of (8) as having the form:

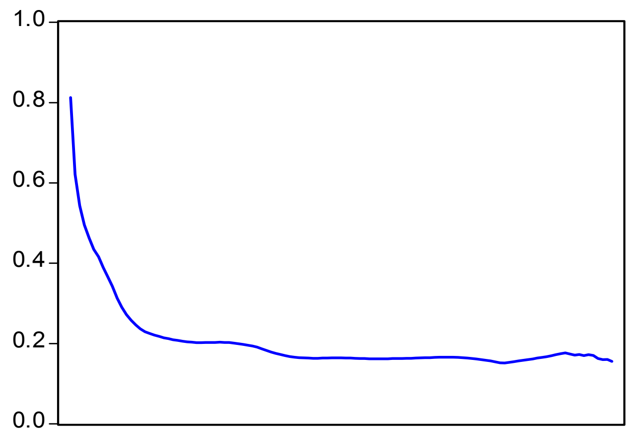
$$\hat{t} = \frac{1}{j}\sum_{i=1}^{j}\left(\hat{e}_i - R_i\right) \qquad (26)$$

where $\hat{e}_i$ is the estimated value of the fund return at observation i: $\hat{e}_i = e(G_i, \hat{\theta})$. G_i is the i[th] Forsythe polynomial, calculated over the entire range of *R* values, after the methodology presented earlier.

It is important to note that for the calculation of the differences and the values of the OMD function, **the series of benchmark returns must be sorted in ascending order**, so as every new value of the benchmark return to represent a new threshold of aversion to risk.

Proceeding this way, we obtain the following plot of the OMD function – see figure 5:

Figure 5 – OMD



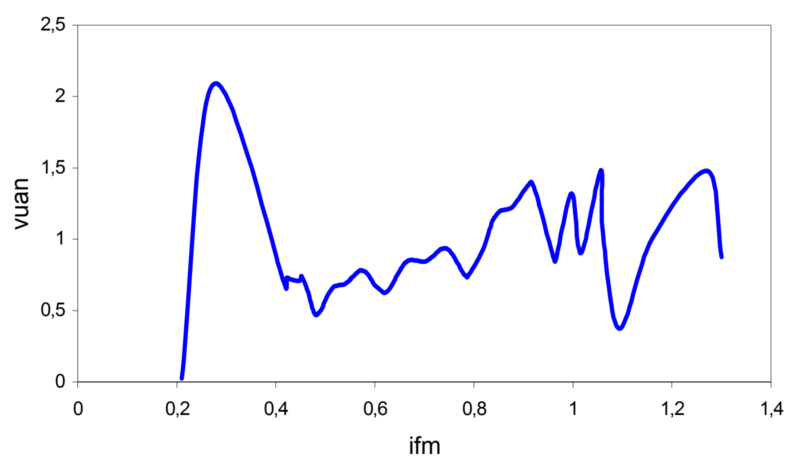
On the horizontal axis, we have the IFM series (taken as benchmark), and on the vertical axis is represented the COMD values, in percents.

As one can see from the graph, *t(P)* is positive over the entire set of the benchmark values. We can therefore conclude that the fund manifested superiority over the market for the period under analyse, in terms of any risk averse investor, no matter his degree of aversion to risk. The only restriction we have to put in order to validate this conclusion is that the investors have a concave and increasing utility function. The conclusion is valid for all investors who enter this class because, as it was demonstrated in Section 2.3., the equivalent margin is the weighted average of OMD (*t(P)*) schedules, at points *P* (each one representing a level of risk aversion, more exactly, the level of the benchmark return he requires the fund manager to equal), and if *t(P)* is greater than 0 for all *P*, then the equivalent margin will also be greater than 0, meaning that the fund is judged to be superior by all the investors).

The OMD curve, being uniformly positive, is preferred by both less and more risk averse investors, but a slightly downward trend can be detected for the lower values of the benchmark, which means that, though, the more risk averse investors are more likely to prefer the fund than the less risk averse ones. However, we should be a little circumspect concerning the downward trend for the first values of the function, because the OMD schedule was constructed as a running mean: for the first values in the series, the running mean was computed from few values of the differences between the fund return and the benchmark return, which means that it is less significant. The number of observation increases with the values for the benchmark return, since the series have been first ascending ordered by the benchmark realisations, and the more observations from which the OMD was calculated we have, the more significant the OMD is. I shall return to this aspect just after two paragraphs.

As specified in Section 2.1., we could as well see the OMD as an area, respectively the *average* area between the regression of the fund return on the benchmark return and the benchmark return itself. Figure 6 presents the respective area. Note that the area in the figure is not exactly the OMD, since, as underlined before, the OMD is an average area.

Figure 6 – the fund return regressed against the benchmark return. The average area between the curve and the Ox axis represents the OMD



Graph 6 shows that for all the benchmark returns, the fund returns were above them. The considered area is positive for the entire set of benchmark returns, which means that the average area up to every point must be positive, too, which leads to the conclusion already spelled: that the fund is judge to have been superior by all the investors, no matter their appetite for risk.

We conclude that **the fund was OMD dominant over the market** (represented by the benchmark), though there have been points and short periods when the fund return fell under the benchmark return (as one can see from table A1 in the annex). But, overall, these falls under the benchmark had not an important effect on the long run performance of the fund.

The inconvenient caused by the fact that for the computation of the first values for OMD there are few observations available can be removed in several ways; Bowden (2000) suggests (but he does not applies anything) using a bootstrapping method or a Baysian approach. In the present paper, I try to implement a rather simpler methodology, based on a exponentially weighting of the differences between the fund returns and the benchmark return. The difficulty consists in choosing the value of the weighting coefficient. The results and the interpretation may be dramatically different according to the different values specified for this coefficient.

The computation of the exponentially weighted ordered mean difference is similar to that of the OMD, just that a weighting coefficient is introduced in the formula. One will have:

$$EWOMD = \frac{1}{j}\sum_{i=1}^{j}\left(r_i - R_i\right)\cdot \delta^{n-i}, \qquad (27)$$

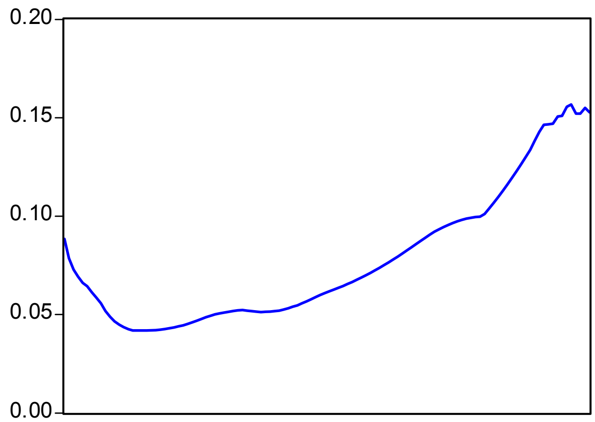
where, as in the initial OMD formula, *r* and *R* are the returns on the fund and on the benchmark, respectively, and δ is the weighting coefficient, which must be between 0 and 1. The weighting coefficient I used in the present paper was calculated from the initial (un-weighted) values of the OMD and from the series of differences between the fund return and the benchmark return, after the following formula:

$$\delta_j = \frac{OMD_j - (r_{j-1} - R_{j-1})}{OMD_{j-1} - (r_{j-1} - R_{j-1})}. \qquad (28)$$

The value of δ that enters formula (20) is simply the average of all the δ_j, with j =1, 2,…n. In our case, n is 117. The idea for this formula for the weighting coefficient comes from the old exponentially weighted moving average models that were used to estimate the volatility on the financial markets.

Proceeding this way, I obtained a δ of 0.98333, that led to the following graph of the EWOMD:

Figure 7 – EWOMD



Of course, the computation of the weighting coefficient is rather arbitrary, one could find a better value for δ. Also, the bootstrapping procedure or the bayesian inference using Monte Carlo simulations – methods suggested in Bowden (2000) – could lead to better results.

Anyway, the OMD and the EWOMD will not produce opposite results in what concerns the judgement of the superiority of the fund over the market; only the judgement in terms of kinds of investors that are more likely to be satisfied by the fund performances relative to the market may change. In our case, it is obvious that, after weighting, the less risk adverse investors seem to be the most satisfied investors.

❑ 3.4. TESTING FOR STOCHASTIC DOMINANCE

We established in Section 2.4. that $t_R(P)>0$ for all P (OMD with the market return as a benchmark) is a necessary, but not a sufficient condition for the fund (r) to second order stochastically dominates the market (R). A sufficient condition would be to demonstrate that the OMD calculated for the market return (with r as a benchmark) to be semi-negative: $t_r(P) \leq 0$.

As noted in Section 2.4., the OMD formula holds if one is wishing to test for stochastic dominance. To do that, we have to compute again the OMD function, but taking the market index return as the dependent variable and the fund return as the benchmark. So, we'll have a regression of the market index (*R*) on the fund return (*r*): $E[R \mid r] = \gamma(r)$. The OMD function with *r* as a benchmark will be:

$$t_r(P) = \frac{1}{F_r(P)} \int_{-\infty}^{P} [\gamma(r) - r] f(r) dr \qquad (29)$$

where: *f(r)* denotes the probability distribution of the fund return, and *F(P)* is the cumulative probability up to point *P*, $F_r(P) = \int_{-\infty}^{P} f(r) dr$. The subscript *r* is used in order to avoid confusion between the two OMD functions (with *R* and *r*, respectively as a benchmark). In section 6, it was demonstrated that if $t_r(P) \leq 0$ for all *P*, then *r* dominates *R* by SSD.

The estimated OMD function for *R* (having *r* as a benchmark) will be:

$$\hat{t}_r = \frac{1}{j} \sum_{i=1}^{j} (\hat{\gamma}_i - r_i) \qquad (30)$$

where $\hat{\gamma}_i$ is the estimated value of the fund return at observation i: $\hat{\gamma}_i = \gamma(G_i, \hat{\lambda})$. G_i is the i[th] Forsythe polynomial, used as an explanatory variable in the regression model. Of course, there are other *G* terms than those used before. The computational procedure is similar to that employed when the OMD function for the fund was introduced.

First, the set of regressors is calculated and introduced in an initial regression equation of *IFM* on *VUAN*. Then, regressors whose coefficients are not significant in terms of the t-test are eliminated. The final regression equation is a 12-term one:

$$IFM_i = \lambda_0 G_{i,0} + \lambda_1 G_{i,1} + \lambda_2 G_{i,2} + \lambda_3 G_{i,3} + \lambda_4 G_{i,4} + \lambda_5 G_{i,5} + \lambda_6 G_{i,6} + \lambda_8 G_{i,8} +$$
$$+ \lambda_{10} G_{i,10} + \lambda_{14} G_{i,14} + \lambda_{16} G_{i,16} + \lambda_{20} G_{i,20} + \varepsilon_i . \qquad (32)$$

The estimation output produced by E-Views is restored in table 8 (page 44).

Table 8: OLS estimation of the regression equation

Dependent Variable: IFM
Method: Least Squares
Included observations: 118
Newey-West HAC Standard Errors & Covariance (lag truncation=4)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
G0	0.688869	0.013991	49.23573	0.0000
G1	0.254860	0.023855	10.68367	0.0000
G2	-0.204722	0.034906	-5.864945	0.0000
G3	-0.510475	0.058354	-8.747823	0.0000
G4	-0.638084	0.131906	-4.837393	0.0000
G5	0.458839	0.208995	2.195454	0.0303
G6	-2.456533	0.456506	-5.381167	0.0000
G8	-6.862339	1.204977	-5.694995	0.0000
G10	-23.29743	6.716903	-3.468478	0.0008
G14	1119.356	577.8509	1.937102	0.0554
G16	29408.48	5847.269	5.029439	0.0000
G20	-12247177	4429749.	-2.764757	0.0067

R-squared	0.617662	Mean dependent var	0.688869
Adjusted R-squared	0.577986	S.D. dependent var	0.183587
S.E. of regression	0.119263	Akaike info criterion	-1.318826
Sum squared resid	1.507710	Schwarz criterion	-1.037061
Log likelihood	89.81071	F-statistic	15.56745
Durbin-Watson stat	1.462238	Prob(F-statistic)	0.000000

As before, we are interested in testing whether the OLS restrictions are satisfied. The principal results are summarised in table 9 (page 45). The figures in parenthesis in the "Value" column are the probability values associated to the value of the calculated statistic, as provided by E-Views. I considered they were necessary because the Newey-West methodology for calculating the standard errors was used.

The table indicates the presence of serial correlation between residuals until the 19th lag and the presence of ARCH effects in the residuals until the 10th lag. The White statistic leads us to accepting the null hypothesis of homoscedasticity, at least at a 5% level of significance (looking at the p-value). However, in the presence of autocorrelation in the residuals series, the estimators will still be unbiased and consistent, but they will no longer be efficient, meaning that the standard errors are no longer valid, and neither the statistics constructed by using these standard errors.

In order to avoid such problems, I used the Newey-West adjustment to correct the standard errors. The truncation lag set by E-Views is the one suggested by Newey and West, respectively the integer of $4(T/100)^{2/9}$ (where T is the number of observations). The effect can be contemplated looking at the level of rejecting a null hypothesis. If we look, for example, at the Breusch-Godfrey test, we can see that, according to the tabulated values, we can accept the alternative hypothesis of serial correlation between residuals until the 18th lag inclusively, but the Newey-West adjusted p-values suggest that the alternative hypothesis of serial correlation is maintained until the 19th lag inclusively. A similar conclusion can be drawn relating the White and the ARCH LM tests.

Table 9 – Summary of statistics for the IFM regression

Test	Distribution	Value	5% critical value
Non-autocorrelation between residuals			
Durbin-Watson	–	1.462238	–
LM (Breusch-Godfrey)	χ^2 (16)	29.73198 (0.019443)	26.30
	χ^2 (18)	30.01623 (0.037289)	28.75
	χ^2 (19)	30.01635 (0.051591)	30.14
	χ^2 (20)	30.03075 (0.069357)	31.41
Homoscedasticity			
White	χ^2 (32)	11.13074	45.91
	R^2	0.094328	–
ARCH LM (Engle)	χ^2 (9)	19.15946 (0.023871)	16.92
	χ^2 (10)	19.15025 (0.038395)	18.31
	χ^2 (11)	19.50315 (0.052638)	19.68
	χ^2 (12)	19.26912 (0.082237)	21.03
Standard normality of residuals			
Skewness	–	0.863407	(0.00)
Kurtosis	–	9.057571	(3.00)
Jarque-Berra	χ^2 (2)	195.0739 (0.0000)	5.99

An interesting observation can de drawn from the correlogram (not presented here): we cannot reject the hypothesis of correlation between residuals until the 17th lag, but the values of the autocorrelation and partial autocorrelation functions appear to be significant at a 5% level only for the first two lags.

As before, the residuals are not normally distributed: the Jarque-Berra statistic is far greater than the critical value of a χ^2 distribution at a 5% significance level, deviations generated especially by the excess kurtosis. But, as stated earlier, this seems not to be too troublesome, since the asymptotic distribution of the estimated parameters can be approximated by their asymptotic distribution in terms of standard normality, and the values of the t and F statistics do not differ significantly.

The independence between the regressors is insured by construction of the model, using the Forsythe polynomials, and the independence between the regressors and the regressors can also be checked if one is looking at the covariance matrix – table 10 below:

Table 10 – Covariance matrix between the residuals and the regressors

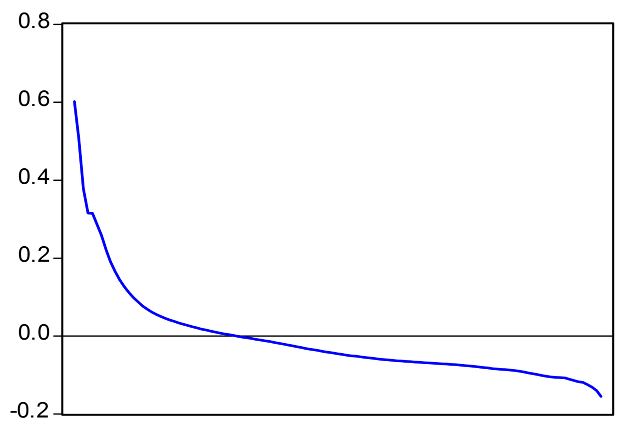
	G0	G1	G2	G3	G4	G5
RESID	1.34E-32	-2.40E-17	-6.98E-18	1.67E-17	4.74E-20	-2.67E-18

Table 10 (continued)

	G6	G8	G10	G14	G16	G20
RESID	1.06E-18	-1.40E-19	1.03E-20	4.47E-22	2.21E-23	9.16E-26

We can therefore compute the OMD function for the market index with VUAN as independent variable, in a similar manner that we proceeded when constructed the OMD function for the fund return. Now, the series are ordered by the VUAN values. The OMD for the IFM is graphically presented in figure 8.

Figure 8 – OMD for IFM



Graph 8 emphasises the regression of the mutual fund index (IFM) (on the vertical axis) on the fund return (on the horizontal axis). As one can see, the OMD is not negative over the entire range of VUAN values. Therefore, **we cannot say that the fund second order stochastically dominated the benchmark** over the entire set of fund returns, though, as it was demonstrated in Section 3.3., it dominated the market in terms of OMD. It means that we are in the situation when the fund is OMD dominant over the Benchmark, but not SSD.

We can say that the fund is dominant in terms of SSD only for the greater values of the fund returns. This finding comes to strengthen the conclusion we drawn after the computation of OMD: the fund was preferred more by the more risk averse investors, those investors who fix lower levels they wish the fund attain.

The same discussion concerning the disadvantage induced by the fact that the OMD is computed only from few observations at the beginning, that has been carried when the OMD for the fund return was computed is valid now, too.

3.5. AN APPROACH TO COINTEGRATION

The OMD is a measure of performance that has a sense if it is computed over a sufficient long period of time; a fund is said to have been superior over the market in terms of OMD (or, OMD dominated the market) if the average area between the regression curve of the fund on the benchmark and the benchmark itself (whose values are taken on the abscissa) is always positive, over the entire set of the benchmark values. There might have been temporary falls of the returns on the fund under the returns on the benchmark, but if the specified area remains positive, the fund is said to have been superior to the market for the analysed period.

This type of assessing performance seems close enough to the cointegration concept in econometrics, which describes the common behaviour of two time series on the long run. As Alexander (2001) points out, "the fundamental aim of cointegration analysis is to detect any common stochastic trends in the price data, and to use these common trends for a dynamic analysis of correlation in returns." In the present section, the cointegration analysis will be made not on yield data, but on return data, for two reasons: (1) because of the atypical situation of the Romanian mutual funds: if, generally, in a market, the yield and the price series are non-stationary, and, after differencing once, the obtained return series is stationary, in the Romanian mutual fund context, the situation is exactly conversely; (2) because the OMD measure is stated in terms of return data. More, algebraically, the OMD measure is a running mean difference between the two series of returns: on the fund and on the benchmark, and this may be seen as an analogy with the cointegration principle, which implies a linear combination between the two series of return.

My aim, in this section, is to make a connection between the OMD and the cointegration principle, in the sense that the result obtained in this section must be consistent with the conclusion drawn when applying OMD.

As specified in Section 2.3, the VUAN and the IFM series are both integrated of order 1, or, formally writing, VUAN ~ I(1) and IFM ~ I(1)[6]. The ADF test result demonstrates this, as can be seen from table 11.

Table 11 – ADF unit root test for VUAN and IFM series

Hypothesis	VUAN	IFM	McKinnon critical values	
I(1) vs. I(0)	-1.721831	-2.075862	1%	-3.4895
I(2) vs. I(1)	-8.475460	-7.039432	5%	-2.8872
			10%	-2.5807

In section 2.3. the residuals series of the regression of *VUAN* on *IFM* (split in several regressors) was shown to be stationary (see table 7, page 37). It follows that the two series are cointegrated.

An Engle-Granger methodology was first tried to find a cointegrating vector between *VUAN* and *IFM*, but the R^2 for the two regressions (on *VUAN* on a constant and *IFM* and the reciprocal) was 0.315224, as one could contemplate in the annex tables A.3. and A.4., which means that two different cointegrating vectors would have resulted. The Johansen methodology removes this disadvantage.

We can apply the Johansen methodology to find a cointegrating vector and then to check to see if the long run relation defines approximately the OMD series. I used a VAR(3) system of two equations: IFM and VUAN:

$$\begin{cases} VUAN_t = \alpha_1 + \beta_{11}VUAN_{t-1} + \beta_{12}VUAN_{t-2} + \beta_{13}VUAN_{t-3} + \beta_{14}IFM_{t-1} + \beta_{15}IFM_{t-2} + \beta_{16}IFM_{t-3} + \varepsilon_{1,t} \\ IFM_t = \alpha_2 + \beta_{21}VUAN_{t-1} + \beta_{22}VUAN_{t-2} + \beta_{23}VUAN_{t-3} + \beta_{24}IFM_{t-1} + \beta_{25}IFM_{t-2} + \beta_{26}IFM_{t-3} + \varepsilon_{2,t} \end{cases}$$

The t statistics display good enough values to validate this model, as one can see from table 12 (page 49), the E-Views output, and from tables A.4. and A.6. in the annex.

[6] The problem of non-stationarity of the dependent variable did not affect the OLS estimation in Section 2.3. since the residuals series is stationary, as shown in the same section.

Table 12 – VAR estimation

Sample(adjusted): 1/24/2000 4/01/2002
Included observations: 115 after adjusting endpoints
Standard errors & t-statistics in parentheses

	VUAN	IFM
VUAN(-1)	-0.220995	-0.052969
	(0.08615)	(0.05761)
	(-2.56529)	(-0.91945)
VUAN(-2)	0.204817	0.065782
	(0.07629)	(0.05102)
	(2.68478)	(1.28946)
VUAN(-3)	0.196593	0.093302
	(0.06740)	(0.04507)
	(2.91671)	(2.07000)
IFM(-1)	0.444879	0.546320
	(0.14812)	(0.09905)
	(3.00358)	(5.51567)
IFM(-2)	0.121431	-0.014198
	(0.16046)	(0.10730)
	(0.75676)	(-0.13231)
IFM(-3)	0.579342	0.121583
	(0.14079)	(0.09415)
	(4.11508)	(1.29143)
C	-0.113444	0.142792
	(0.08491)	(0.05678)
	(-1.33598)	(2.51464)
R-squared	0.565360	0.499676
Adj. R-squared	0.541214	0.471881
Sum sq. resids	3.879313	1.734785
S.E. equation	0.189525	0.126739
Log likelihood	31.70533	77.97988
Akaike AIC	31.82707	78.10162
Schwarz SC	31.99415	78.26870
Mean dependent	0.836723	0.685719
S.D. dependent	0.279808	0.174399
Determinant Residual Covariance		0.000454
Log Likelihood		116.2009
Akaike Information Criteria		116.4443
Schwarz Criteria		116.7785

A **LR test** has been applied to verify if the lag order of the VAR had been chosen correctly. The null hypothesis of 2 lags was rejected with an LR statistic of 44.82642, while the corresponding $\chi^2(4)$ 95% critical values is 9.49. The test statistic was calculated after the formula:

$$LR = -2 \cdot (\ell_2 - \ell_3) = -2 \cdot (93.78769 - 116.2009) = 44.82642 > 9.49,$$

where ℓ_2 and ℓ_3 are the log-likelihood values for VAR(2) and VAR(3), respectively.

A similar test has been carried to test the hypothesis of VAR(3) versus VAR(4). The test indicated the acceptance of the null hypothesis of VAR(3):

$$LR = -2 \cdot (\ell_3 - \ell_4) = -2 \cdot (116.2009 - 116.7436) = 1.0854 < 9.49.$$

The critical value was that for χ^2 with 4 degrees of freedom because there were 4 zero restrictions when passing from a lag order of 2 to a lag order of 3, respectively from one of 3 to one of 4.

The fact that VAR(3) is a better specified model relative to VAR(2) results also when looking at the serial correlation of the residuals series for the two equations in the system. The annex contains four tables (A.4.– A.7.) with the E-View output for the two equations in the two cases (order lag of 2 and order lag of 3 for VAR). Table 13 below presents the main results of testing the hypothesis of serial correlation of the residuals in the four cases.

Table 13 – OLS separate estimation for the VAR equations

	VAR(3)		VAR(2)		Distribution	Critical value
	VUAN	IFM	VUAN	IFM		
DW	1.981266	1.968314	2.198261	2.138932	–	(2.00)
Q(1)	0.0059	0.0100	2.4452	0.6026	$\chi^2(1)$	3.84
Q(2)	0.3807	0.0479	2.4772	0.8041	$\chi^2(2)$	5.99
Q(3)	1.5806	3.2863	6.6508	0.8046	$\chi^2(3)$	7.82
Q(4)	1.8126	5.0729	9.0332	1.3943	$\chi^2(4)$	9.49
BG(2)	0.969355	0.216748	14.07406	4.232258	$\chi^2(2)$	5.99
BG(3)	5.207536	7.043206	14.16348	4.927487	$\chi^2(3)$	7.82
BG(4)	6.387876	15.02648	14.27377	6.733587	$\chi^2(4)$	9.49
BG(6)	7.358067	18.56603	15.08536	7.051775	$\chi^2(6)$	12.59
White	31.6205 [$\chi^2(27)$]	81.92332 [$\chi^2(27)$]	35.87982 [$\chi^2(14)$]	16.12749 [$\chi^2(14)$]	$\chi^2(27)$ $\chi^2(14)$	40.11 23.69
ARCH-LM(2)	4.698600	6.781785	7.482439	3.278474	$\chi^2(2)$	5.99
ARCH-LM(3)	4.744544	7.043206	10.03964	3.637078	$\chi^2(3)$	7.82
ARCH-LM(4)	6.781491	8.752868	10.63870	4.196441	$\chi^2(4)$	9.49

Note: **DW** is the Durbin-Watson statistic, **Q** is the Ljung-Box statistic and **BG** is the Breusch-Godfrey statistic

Actually, the equation for IFM in VAR(2) behaved better than in VAR(3) (residuals series in the IFM series in VAR(3) indicate general heteroscedasticity and autocorrelation at lags of order greater than 4), but overall, as demonstrated before, VAR(3) is a better model than VAR(2).

The Johansen test has been performed assuming no deterministic trend and no intercept in the regression equation. To be sure, I performed the Johansen test specifying no intercept but also having an intercept. The LR test was convincing in removing the intercept:

$LR = -2 \cdot (112.2783 - 114.9594) = 5.3622 < 5.99$, the χ^2 (2) critical value.

The E-Views output for the Johansen test is restored in table 14 below.

Table 14 – Johansen test (E-Views output)

Sample: 1/03/2000 4/01/2002				
Included observations: 114				
Test assumption: No deterministic trend in the data				
Series: VUAN IFM				
Lags interval: 1 to 3				
Eigenvalue	Likelihood Ratio	5 Percent Critical Value	1 Percent Critical Value	Hypothesized No. of CE(s)
0.190826	25.03568	12.53	16.31	None **
0.007839	0.897136	3.84	6.51	At most 1
*(**) denotes rejection of the hypothesis at 5%(1%) significance level				
L.R. test indicates 1 cointegrating equation(s) at 5% significance level				
Unnormalized Cointegrating Coefficients:				
VUAN	IFM			
-0.845297	**1.029964**			
0.019400	0.109297			
Normalized Cointegrating Coefficients: 1 Cointegrating Equation(s)				
VUAN	IFM			
1.000000	**-1.218465**			
	(0.03033)			
Log likelihood	112.2783			

One cointegrating relation between *VUAN* and *IFM* series was found. The long run equilibrium relationship between the two returns is given by:

$$\underline{VUAN - 1.218465 \cdot IFM = 0}.$$

This means that, on the long run, the fund return was 1.22 times greater than the benchmark return. This result is consistent with the previous conclusion (Section 3.3.): the fund has been superior over the market for the analysed period.

We could investigate more aspects relating the fund behaviour if we construct the error correction model (ECM). The E-Views output for the ECM is restored in table 14.

Table 14 – ECM estimation

Sample(adjusted): 1/31/2000 4/01/2002		
Included observations: 114 after adjusting endpoints		
Standard errors & t-statistics in parentheses		
Cointegrating Eq:	CointEq1	
VUAN(-1)	1.000000	
IFM(-1)	-1.218465	
	(0.03033)	
	(-40.1729)	
Error Correction:	D(VUAN)	D(IFM)
CointEq1	-0.741434	0.058970
	(0.16147)	(0.10821)
	(-4.59176)	(0.54494)
D(VUAN(-1))	-0.465839	-0.196431
	(0.13235)	(0.08870)
	(-3.51979)	(-2.21461)
D(VUAN(-2))	-0.234239	-0.152625
	(0.10593)	(0.07099)
	(-2.21129)	(-2.14990)
D(VUAN(-3))	-0.027458	0.001851
	(0.07063)	(0.04734)
	(-0.38875)	(0.03910)
D(IFM(-1))	-0.496704	-0.278560
	(0.20894)	(0.14003)
	(-2.37723)	(-1.98929)
D(IFM(-2))	-0.408305	-0.193728
	(0.17825)	(0.11946)
	(-2.29065)	(-1.62170)
D(IFM(-3))	0.142931	-0.147749
	(0.14665)	(0.09828)
	(0.97464)	(-1.50330)
R-squared	0.637885	0.239768
Adj. R-squared	0.617579	0.197138
Sum sq. resids	3.904379	1.753632
S.E. equation	0.191022	0.128020
Log likelihood	30.56469	76.18805
Akaike AIC	30.68750	76.31085
Schwarz SC	30.85551	76.47887
Mean dependent	-0.008551	-0.003381
S.D. dependent	0.308897	0.142875

Looking at the coefficients of the lagged values of *IFM* in the *VUAN* equation and vice-versa, we can see that for the first two lags they are significant, which means that both *IFM* Granger caused *VUAN* and *VUAN* Granger caused *IFM*. This not an unexpected result since Capital Plus enters the composition of IFM.

The adjustment coefficients (-0.7414 and 0.0589) indicate a rather slow speed of adjustment to the long run equilibrium. The fact that they are of contrary signs, is consistent with the theory.

❑ CONCLUSIONS

This paper presented the ordered mean difference, which is a measure of portfolio performance introduced by Roger Bowden by a series of articles. The ordered mean difference (OMD) is derived from the equivalent margin formula, which represents the penalty that is necessary to make an investor reconsider his participation in a mutual fund. The biggest the penalty, the more performant the funs is considered to be. OMD is a special case of the equivalent margin, namely when the utility function has the form of a short put. The investor is interested in obtaining a fixed level of return, being indifferent at what happens above that level and negatively exposed when the return is below that level. The "exercise price" is the threshold he imposes, and it is established in accordance with his appetite for risk. The more risk averse the investor is, the lower level he fixes.

The idea of this type of utility function is older: it was introduced by Robert C. Merton as an alternative for the classical assumption of the fund return being a linear function of the benchmark return. On the other hand, the put-like utility function is the converse of the poverty gap function, introduces by Davisdon and Duclos (2000), which represents how more the investor has until he gets his threshold. These two authors demonstrated that the second order stochastic dominance condition can be written as to show the average poverty gap up to the given poverty threshold.

Returning to OMD, Bowden (2000) showed that the equivalent margin is the weighted average of a set of OMD's. This means that the equivalent margin is a measure of performance that is independent of the degree of aversion to risk of the investor. It was demonstrated that if OMD is positive over the entire range of values of the benchmark, then the equivalent margin also positive. OMD is constructed as a running mean of the differences between the fund return and the benchmark return, with the observations sorted ascending after the benchmark values, and represents the average area between the regression curve of the fund on the benchmark and the benchmark itself. If this area is always positive, the fund is considered to have been superior over the market in the analysed period.

OMD can be used to check for stochastic dominance: if the inverse regression, of the market index on the fund return, taken as a benchmark is negative, then the fund second order stochastically dominated the market.

The empirical part of this paper employed a regression of the fund return on a set of regressors that were obtained by using the Forsythe polynomials, which split the benchmark series in several series. The advantage of using these explanatory variables instead of defining a quadratic or a non-linear regression is that doing so, we insure against the collinearity between regressors. More, the coefficient of determination proved to be larger.

The fund, Capital Plus, proved to be OMD dominant over the market, but not SSD. As the benchmark was considered the mutual fund index, which is an index constructed in the same manner as an exchange index. A large comment about the superiority of the fund was made in Sections 3.3. and 3.4.

The final section of the paper made an approach to the cointegration theory in econometrics and it was there where I showed that the conclusion of superiority of the fund over the market is sustained by the cointegrating relation that was found between the fund return and the benchmark return. The idea of associating OMD with the cointegration theory comes from the fact that they both have a long run perspective. The OMD measures allows the fund return to temporary fall below the market index without affecting the general result if these falls were not very drastic.

Econometric tests have been applied in constructing and verifying the regression models that were used.

❑ ANNEXES

Table A.1. Data used: the fund (VUAN) and the benchmark (IFM) series of returns (%)*

Date	VUAN (%)	IFM (%)
1/03/00	2.102349	0.273778
1/10/00	1.187074	1.060852
1/17/00	0.115866	1.094163
1/24/00	1.222423	0.879864
1/31/00	1.372017	0.845694
2/07/00	1.113774	0.840499
2/14/00	1.924149	1.056570
2/21/00	0.937073	0.817404
2/28/00	1.023243	0.919398
3/06/00	1.710491	0.930866
3/13/00	1.140935	0.858966
3/20/00	1.141106	0.854243
3/27/00	1.237831	1.159580
4/03/00	1.203592	0.883068
4/10/00	1.271080	0.997416
4/17/00	1.099789	0.841937
4/24/00	1.173868	0.966209
5/01/00	1.324262	0.884884
5/08/00	1.318945	0.838033
5/15/00	1.094675	0.914215
5/22/00	1.012584	0.873304
5/29/00	1.014022	0.963179
6/05/00	0.917800	0.640486
6/12/00	0.784403	0.424300
6/19/00	0.851616	1.300761
6/26/00	1.426015	1.271949
7/03/00	0.711253	1.016490
7/10/00	2.069419	0.745379
7/17/00	0.745548	0.422375
7/24/00	0.921046	0.424001
7/31/00	0.870437	0.452555
8/07/00	0.810627	0.464176
8/14/00	0.762606	0.465513
8/21/00	0.705349	0.676120
8/28/00	0.516360	0.771493
9/04/00	0.727328	0.661988
9/11/00	0.701878	0.644184
9/18/00	0.707015	0.661197
9/25/00	0.707030	0.651411
10/02/00	0.865223	0.578969
10/09/00	0.700946	0.618567
10/16/00	0.793419	0.591970
10/23/00	0.816149	0.736072
10/30/00	0.795171	0.742869
11/06/00	0.798403	0.732870
11/13/00	0.683640	0.786777
11/20/00	0.655584	0.693830
11/27/00	0.683880	0.706091
12/04/00	0.771648	0.595412
12/11/00	0.554817	0.754244
12/18/00	0.652075	0.699735
12/25/00	0.702215	0.667619
1/01/01	0.643333	0.552119
1/08/01	0.701801	0.571947
1/15/01	0.781250	0.781813
1/22/01	0.753171	0.669036
1/29/01	0.852459	0.723302
2/05/01	0.775899	0.759319
2/12/01	0.838746	0.720575
2/19/01	0.648354	0.715612
2/26/01	0.796745	0.638349
3/05/01	0.697948	0.652183
3/12/01	0.830898	0.719631
3/19/01	0.915152	0.749481
3/26/01	0.808371	0.652166
4/02/01	0.810030	0.731756
4/09/01	1.130582	0.724820
4/16/01	0.798531	0.728187
4/23/01	0.895191	0.735171

Date	VUAN (%)	IFM (%)
4/30/01	0.828361	0.769719
5/07/01	0.868279	0.775381
5/14/01	0.853084	0.764469
5/21/01	0.899453	0.762286
5/28/01	0.845915	0.765235
6/04/01	0.944141	0.716937
6/11/01	0.909226	0.740630
6/18/01	0.853028	0.778314
6/25/01	0.721321	0.720799
7/02/01	0.872473	0.704278
7/09/01	0.764019	0.699843
7/16/01	0.736767	0.620439
7/23/01	0.770432	0.635012
7/30/01	0.799775	0.699807
8/06/01	0.796924	0.627576
8/13/01	0.801026	0.635769
8/20/01	0.849702	0.598962
8/27/01	0.842543	0.699514
9/03/01	0.622400	0.588931
9/10/01	0.753017	0.582242
9/17/01	0.563878	0.568744
9/24/01	0.753152	0.551796
10/01/01	0.470906	0.571301
10/08/01	0.803015	0.561945
10/15/01	0.702325	0.591213
10/22/01	0.729715	0.545229
10/29/01	0.743661	0.591671
11/05/01	0.722263	0.569167
11/12/01	0.713925	0.550989
11/19/01	0.712001	0.540970
11/26/01	0.710081	0.545239
12/03/01	0.701982	0.539036
12/10/01	0.703231	0.498265
12/17/01	0.673924	0.522566
12/24/01	0.651239	0.549037
12/31/01	0.713232	0.718306
1/07/02	-0.062750	0.210378
1/14/02	1.195993	0.548565
1/21/02	0.602748	0.492539
1/28/02	0.613821	0.420315
2/04/02	0.604238	0.621109
2/11/02	0.600609	0.516521
2/18/02	0.628749	0.499956
2/25/02	0.627687	0.504186
3/04/02	0.626620	0.479986
3/11/02	0.639701	0.501477
3/18/02	0.632822	0.447634
3/25/02	0.441587	0.468799
4/01/02	0.247649	0.494434

* The blue figures indicate observations where the fund return fell under the benchmark return

Table A.2. Why not Engle-Granger (Section 3.5.; page 48)

Dependent Variable: VUAN
Method: Least Squares
Date: 06/14/02 Time: 18:33
Sample: 1/10/2000 4/01/2002
Included observations: 117
VUAN=C(1)+C(2)*IFM

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.214249	0.087932	2.436546	0.0164
C(2)	0.894414	0.122929	7.275862	0.0000

R-squared	0.315224	Mean dependent var	0.833556
Adjusted R-squared	0.309270	S.D. dependent var	0.287177
S.E. of regression	0.238673	Akaike info criterion	-0.010499
Sum squared resid	6.550951	Schwarz criterion	0.036718
Log likelihood	2.614173	F-statistic	52.93817
Durbin-Watson stat	1.924629	Prob(F-statistic)	0.000000

Table A.3. Why not Engle-Granger (Section 3.5.; page 48)

Dependent Variable: IFM
Method: Least Squares
Date: 06/14/02 Time: 18:33
Sample: 1/10/2000 4/01/2002
Included observations: 117
IFM=C(1)+C(2)*VUAN

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.398641	0.042686	9.338814	0.0000
C(2)	0.352437	0.048439	7.275862	0.0000

R-squared	0.315224	Mean dependent var	0.692416
Adjusted R-squared	0.309270	S.D. dependent var	0.180269
S.E. of regression	0.149822	Akaike info criterion	-0.941796
Sum squared resid	2.581351	Schwarz criterion	-0.894580
Log likelihood	57.09508	F-statistic	52.93817
Durbin-Watson stat	1.173491	Prob(F-statistic)	0.000000

Table A.4. VUAN equation in VAR(3) (Section 3.5.)

Dependent Variable: VUAN
Method: Least Squares
Date: 06/15/02 Time: 13:56
Sample(adjusted): 1/24/2000 4/01/2002
Included observations: 115 after adjusting endpoints
VUAN=C(1)*VUAN(-1)+C(2)*VUAN(-2)+C(3)*VUAN(-3)+C(4)*IFM(-1)+C(5)*IFM(-2)
+C(6)*IFM(-3)+C(7)

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	-0.220995	0.086148	-2.565286	0.0117
C(2)	0.204817	0.076288	2.684777	0.0084
C(3)	0.196593	0.067402	2.916706	0.0043
C(4)	0.444879	0.148116	3.003579	0.0033
C(5)	0.121431	0.160462	0.756756	0.4508
C(6)	0.579342	0.140785	4.115084	0.0001
C(7)	-0.113444	0.084915	-1.335978	0.1844

R-squared	0.565360	Mean dependent var	0.836723
Adjusted R-squared	0.541214	S.D. dependent var	0.279808
S.E. of regression	0.189525	Akaike info criterion	-0.429658
Sum squared resid	3.879313	Schwarz criterion	-0.262575
Log likelihood	31.70533	F-statistic	23.41362
Durbin-Watson stat	1.981266	Prob(F-statistic)	0.000000

Table A.6. VUAN equation in VAR(2) (Section 3.5.)

Dependent Variable: VUAN
Method: Least Squares
Date: 06/15/02 Time: 13:53
Sample(adjusted): 1/17/2000 4/01/2002
Included observations: 116 after adjusting endpoints
VUAN=C(1)*VUAN(-1)+C(2)*VUAN(-2)+C(3)*IFM(-1)+C(4)*IFM(-2)+C(5)

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	-0.040984	0.089949	-0.455639	0.6495
C(2)	0.113911	0.077813	1.463895	0.1460
C(3)	0.256640	0.163333	1.571271	0.1190
C(4)	0.756188	0.148735	5.084111	0.0000
C(5)	0.065965	0.094647	0.696956	0.4873

R-squared	0.407989	Mean dependent var	0.830508
Adjusted R-squared	0.386655	S.D. dependent var	0.286516
S.E. of regression	0.224389	Akaike info criterion	-0.108726
Sum squared resid	5.588889	Schwarz criterion	0.009963
Log likelihood	11.30609	F-statistic	19.12410
Durbin-Watson stat	2.198216	Prob(F-statistic)	0.000000

Table A.6. (Section 3.5.)

Dependent Variable: IFM
Method: Least Squares
Date: 06/15/02 Time: 14:38
Sample(adjusted): 1/24/2000 4/01/2002
Included observations: 115 after adjusting endpoints
IFM=C(1)*VUAN(-1)+C(2)*VUAN(-2)+C(3)*VUAN(-3)+C(4)*IFM(-1)+C(5)*IFM(-2)
+C(6)*IFM(-3)+C(7)

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	-0.052969	0.057609	-0.919451	0.3599
C(2)	0.065782	0.051015	1.289455	0.2000
C(3)	0.093302	0.045074	2.069998	0.0408
C(4)	0.546320	0.099049	5.515673	0.0000
C(5)	-0.014198	0.107305	-0.132312	0.8950
C(6)	0.121583	0.094146	1.291428	0.1993
C(7)	0.142792	0.056784	2.514635	0.0134

R-squared	0.499676	Mean dependent var	0.685719
Adjusted R-squared	0.471881	S.D. dependent var	0.174399
S.E. of regression	0.126739	Akaike info criterion	-1.234433
Sum squared resid	1.734785	Schwarz criterion	-1.067350
Log likelihood	77.97988	F-statistic	17.97672
Durbin-Watson stat	1.968314	Prob(F-statistic)	0.000000

Table A.7. (Section 3.5.)

Dependent Variable: IFM
Method: Least Squares
Date: 06/15/02 Time: 14:39
Sample(adjusted): 1/17/2000 4/01/2002
Included observations: 116 after adjusting endpoints
IFM=C(1)*VUAN(-1)+C(2)*VUAN(-2)+C(3)*IFM(-1)+C(4)*IFM(-2)+C(5)

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.000470	0.051802	0.009079	0.9928
C(2)	0.102081	0.044813	2.277943	0.0246
C(3)	0.586036	0.094064	6.230195	0.0000
C(4)	0.020452	0.085657	0.238771	0.8117
C(5)	0.180835	0.054507	3.317624	0.0012

R-squared	0.489740	Mean dependent var	0.689240
Adjusted R-squared	0.471352	S.D. dependent var	0.177732
S.E. of regression	0.129226	Akaike info criterion	-1.212362
Sum squared resid	1.853628	Schwarz criterion	-1.093672
Log likelihood	75.31698	F-statistic	26.63406
Durbin-Watson stat	2.138932	Prob(F-statistic)	0.000000

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