ACADEMY OF ECONOMIC STUDIES
DOCTORAL SCHOOL OF FINANCE AND BANKING

ANALYZING ASYMMETRIC DEPENDENCE IN EXCHANGE RATES USING COPULAS

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Bucharest, July 2010

Contents

1. Problem overview

- Asymmetric dependence in exchange rates
- Switching regimes of dependence parameters over time
- Empirical evidence of *leverage effect*
- Extreme events in the tails of distribution
- Basel II amendment
- Large-scale utilization of *Value-at-Risk* models in financial and banking system
- Stylized facts in exchange rates returns

2. Objectives

General objective:

•In this paper I aimed to analyze the asymmetric dependence in four exchange rates from Central and Eastern Europe in order to choose the most suitable copulas to improve the accuracy of VaR models. For this purpose I split the engaged copulas in two categories: Elliptical plus Plackett and Archimedean copulas. Regarding this goal I proposed the decomposition of initial portfolio in other three bivariate portfolios to use the copulas that provide the lowest negative log-likelihood values.

Intermediary objectives:

•Modeling exchange rates returns with ARMA-GJR approach to obtain filtered residuals

•Using Extreme Value Theory to model the tails of standardized residuals distribution

•Estimating the parameters for large portfolio and analyzing conditional dependence between portfolio assets using Canonical Vine copula

•Estimating the parameters for each bivariate portfolio and chose the best copula by information criteria

•Using Monte Carlo simulation to estimate in-sample and forecast out-of-sample risk measures

•Backtesting the results with Bernoulli and Kupiec methods.

3. Literature review

- Rockinger and Jondeau (2001) used Plackett copula to analyze the dependence among S&P500, Nikkei 225 and some European stock indices.
- Patton (2001) established the background for conditional copula in order to allow first and second order moments of distribution function to vary over time.
- Embrechts and Dias (2004) used ARMA-GARCH model to filter the residuals for the estimation of copula parameters. The analyzed series were spot rates of Japanese Yen and German Mark against US Dollar.
- Hotta, Lucas and Palaro (2006) estimated Value-at-Risk using an ARMA-GARCH model to filter returns, while the marginal distributions and dependence structure were modeled with a GPD approach, respectively with a Gumbel copula. They analyzed a portfolio composed of Bovespa and Merival indices.
- Patton (2006) used conditional Gaussian and Symetrized Joe-Clayton copulas to analyze the asymmetric distribution between German Mark and Yen against US Dollar.
- Aas (2007) proposed a Canonical Vine copula model to decompose the portfolio of four indices in bivariate pairs. Estimated parameters were compared with those resulted from bivariate and four-dimensional Student's copula.
- Chollete, Heineny and Valdesogo (2008) used Canonical Vine copulas to model the asymmetric dependence between financial returns. Heineny and Valdesogo (2009) introduce a Canonical Vine autoregressive copula to model dynamic dependence between more than 30 assets.

4. Methodology: ARMA-GJR Model

- conditional mean equation

$$y_t = c + \sum_{i=1}^{m} \phi_i y_{t-i} + \sum_{j=1}^{n} \theta_j \varepsilon_{t-j} + \varepsilon_t$$

- conditional variance equation

$$\sigma_t^2 = \omega + \sum_{i=1}^{P} \beta_i \sigma_{t-i}^2 + \sum_{j=1}^{Q} \alpha_j \varepsilon_{t-j}^2$$

- Independently Glosten, Jagannathan and Runkle (1993) and Zakoian (1994) introduced an indicator function to incorporate leverage effect (Black, 1976) into the variance:

$$\sigma_t^2 = \omega + \sum_{i=1}^{P} \beta_i \sigma_{t-i}^2 + \sum_{j=1}^{Q} \alpha_j \varepsilon_{t-j}^2 + \sum_{j=1}^{Q} \gamma_j \varepsilon_{t-j}^2 I_{t-j}$$

$$I_{\mathrm{t-j}} = \begin{cases} 1 & \varepsilon_{\mathrm{t-j}} < 0 \text{ (bad news)} \\ 0 & \varepsilon_{\mathrm{t-j}} \geq 0 \text{ (good news)} \end{cases}$$

$\mathrm{z_t} = \varepsilon_{\mathrm{t}} / \sigma_t$ is independent and identically distributed

4. Methodology: Generalized Pareto Distribution

***Peak-over-threshold* approach**

- Given a random vector $X = (X_1, \ldots, X_n)$ with a distribution function $\varphi(x) = \Pr(X \le x)$ and considering a threshold value υ, excesses over υ are defined as: $E = X - \upsilon$.

  Thus the distribution function of excesses is:

$$\varphi_v(x) = \Pr(X - \upsilon \le x \mid X > \upsilon)$$

Independently Balkema and de Haan (1974) and Pickands (1975) showed that for $\upsilon \to \infty$, the distribution function of the exceedances may be approximated by the Generalized Pareto Distribution (GPD):

$$G_{\xi,\sigma,\beta} = \begin{cases} 1 - \left(1 + \xi \dfrac{E-\beta}{\sigma}\right)^{-\frac{1}{\xi}}, & \text{if } \xi \neq 0 \\ 1 - e^{-\frac{(E-\beta)}{\sigma}}, & \text{if } \xi = 0 \end{cases}$$

where ξ is the tail index, β is location parameter and σ represents the scale parameter; parameter

4. Methodology: Copula models

Copula definitions

Sklar's Theorem (1959). *If F is a n-dimensional joint distribution function with the continuous marginal distributions* $F_1, \dots, F_n$ *then there exist a unique n-copula* $C\,[0,1]^n \rightarrow [0,1]$ *such that:*

$$F(x_1, \dots, x_n) = C\big(F_1(x_1), \dots, F_n(x_n)\big).$$

In 1996, Sklar defined copula as a function that links a multidimensional distribution to its one dimensional margins.

Inversely, if there are known the distribution functions for the *n*-dimensional joint distribution and marginal distributions, then the copula is given by the following formula:

$$C(u_1, \dots, u_n) = F\big({F^{-1}}_1(u_1), \dots, {F^{-1}}_n(u_n)\big).$$

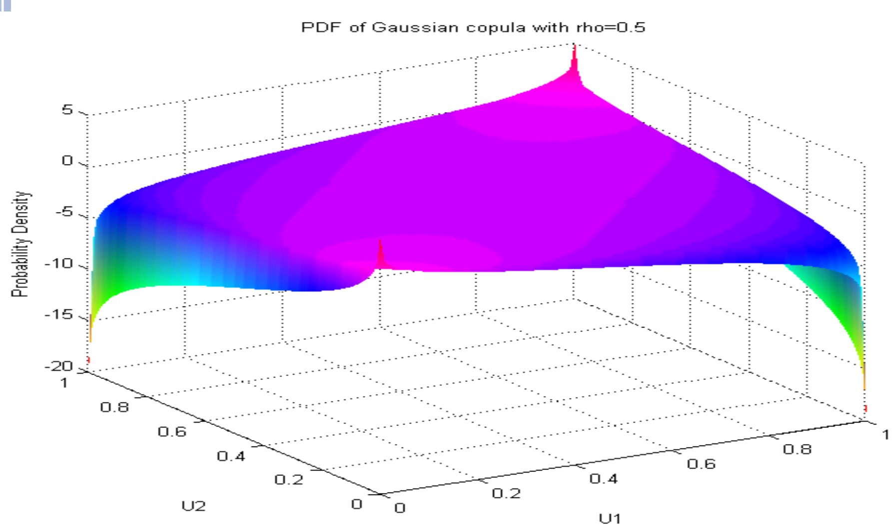
4. Methodology: Bivariate copula examples

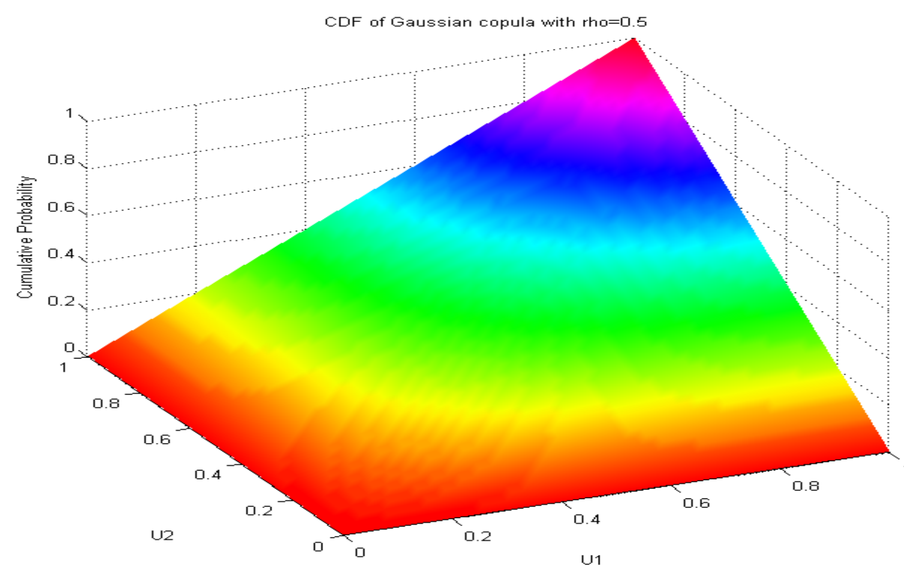
Elliptical copulas

Bivariate Gaussian copula

$$C_{\rho}^{Gauss}(u_1, u_2) = \Phi_{\rho}\left(\Phi^{-1}(u_1), \Phi^{-1}(u_2)\right)$$

where: - Φ_p represents the standard bivariate normal distribution;
- *p* represents the dependence parameter;
- Φ^{-1}(u) represents the inverse of the normal cumulative distribution function.





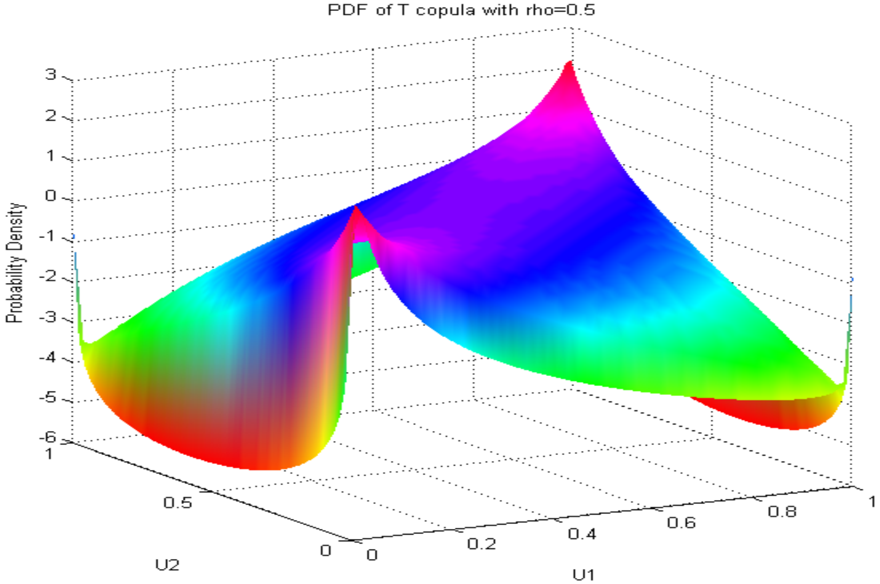
4. Methodology: Bivariate copula examples

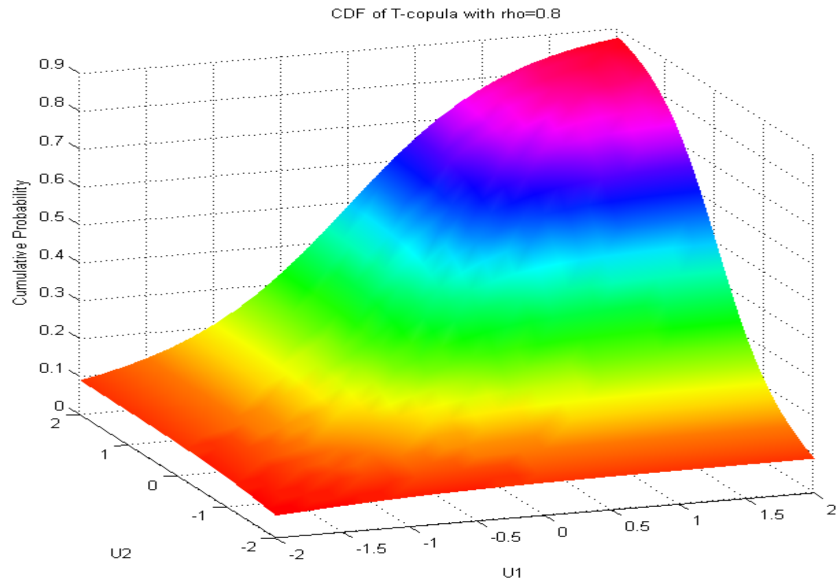
Elliptical copulas

Bivariate Student copula

$$C_{\rho,v}^{Student}(u_1, u_2) = T_{\rho,v}(T_v^{-1}(u_1), T_v^{-1}(u_2))$$

where: - $T_{\rho,\upsilon}$ represents the standard bivariate Student distribution;
- υ represents the dependence parameter;
- $T_\upsilon^{-1}(u)$ represents the inverse of the Student cumulative distribution function.





4.Methodology: Bivariate copula examples

Archimedean copulas

Definition. *Given a continuous function φ from [0,1] onto [0,∞] , strictly decreasing and convex, such that φ(1)=0 and* $\varphi^{[-1]}$ *is a pseudo-inverse of φ:*

$$\varphi^{[-1]}(t) = \begin{cases} \varphi^{[-1]}(t), if\ 0 \leq t \leq \varphi(0) \\ 0, \quad if\ t \geq \varphi(0) \end{cases}$$

then the general form of an Archimedean copula with a generator function φ can be defined as following:

$$C(u, v) = \varphi^{[-1]}(\varphi(u) + \varphi(v))$$

This family of functions is called Archimedean due to unaccomplished *infinitesimality* property between its elements. Thus the Archimedean copulas have not infinite elements.

4.Methodology: Bivariate copula examples

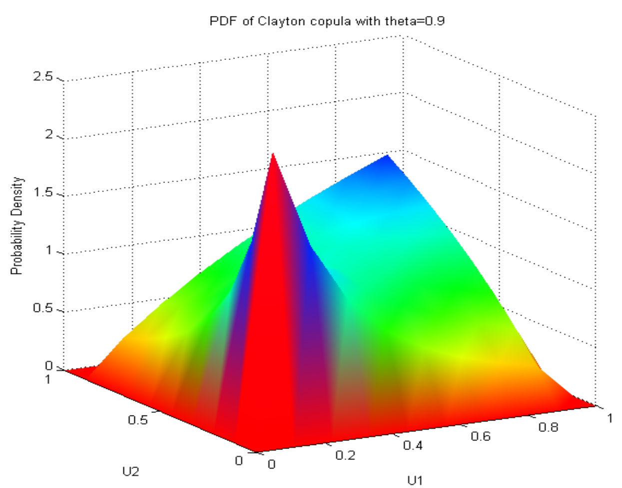
Archimedean copulas

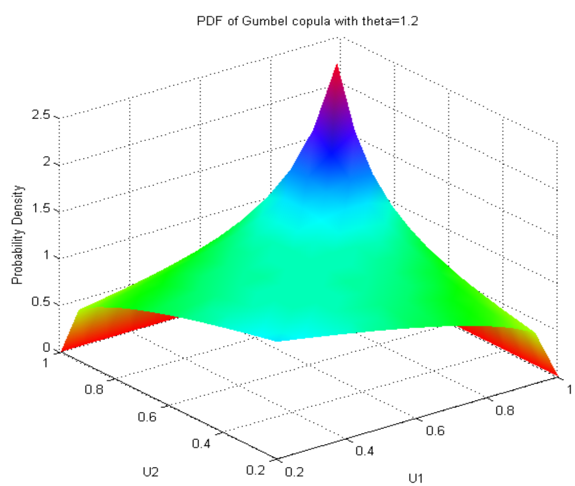
Clayton copula (1978): $$C_\theta^{Clayton}(u,v) = \max\left(\left[u^{-\theta} + v^{-\theta} - 1\right]^{-\frac{1}{\theta}}, 0\right), \theta \in [-1, \infty)$$

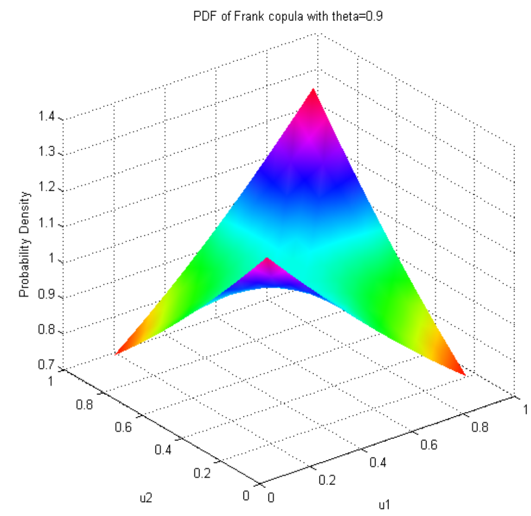
Gumbel copula (1960): $$C_\theta^{Gumbel}(u,v) = \exp\left(-\left[(-\ln u)^{\theta} + (-\ln v)^{\theta}\right]^{\frac{1}{\theta}}\right), \theta \in [1, \infty)$$

Frank copula (1979): $$C_\theta^{Frank}(u,v) = -\frac{1}{\theta}\ln\left(1 + \frac{\left(e^{-\theta u} - 1\right)\left(e^{-\theta v} - 1\right)}{e^{-\theta} - 1}\right), \theta \in \boldsymbol{R}$$

where θ is the dependence parameter.







4. Methodology: Copula models

Copula-GARCH

Patton (2006) extended for the first time (to my knowledge) the Sklar's theorem to conditional distribution

$$\theta_t = \Lambda\left(\omega + \beta\theta_{t-1} + \alpha\frac{1}{m}\sum_{j=1}^{m}\left|u_{t-j} - v_{t-j}\right|\right)$$

where θ represents the dependence parameter, Λ is a transformation, ω is a constant and β an autoregressive term, α denotes the parameter of forcing variable, while *m* is the window length.

Canonical Vine Copula

Given a three dimensional joint distribution function:

$$f(y_1, y_2, y_3) = f(y_1) \cdot f(y_2|y_1) \cdot f(y_3|y_2, y_1)$$

A canonical copula vine model can be defined as:

$$c(y_1, y_2, y_3) = c_{23|1}\left(F_{2|1}(y_2|y_1), F_{3|1}(y_3|y_1)\right) c_{12}\left(F_1(y_1), F_2(y_2)\right) c_{13}\left(F_1(y_1), F_3(y_3)\right)$$

4. Methodology: Quantitative risk measures

Value-at-Risk: $-(Z_\alpha\sigma + \mu)X$ where μ and σ are the sample mean, respectively sample variance, Z_α is α% quartile and *X* denotes the value of an asset or portfolio.

Model's limitations:

- doesn't refer to a potential size of loss if the VaR's limits are exceeded;
- doesn't provide a satisfactory distinction between "good" risks and "bad" risks (Dembo and Freeman, 2001);
- is not a *coherent* measure of risk (Arztner, 1997);

Semi-Variance (Markowitz,1959): $Semi - Variance = E\left(\left(min\left(0, R - E(R)\right)\right)^2\right)$

Regret (Dembo and Freeman, 2001): $Regret = -E\left(min(0, R - Benchmark\_Return)\right)$

Conditional Value-at-Risk (Arztner, 1997): $\text{Conditional VaR}(\alpha) = \mathrm{E}(\mathrm{R}|\mathrm{R} > VaR)$

Conditional Value-at-Risk is a coherent measure of risk.

5.Data and results: Input data

- Four currencies from Central and Eastern Europe against European currency: EUR/CZK, EUR/HUF, EUR/PLN and EUR/RON
- Analyzed period: 5/2/1999- 4/2/2010
- Each series contains 2871 observations of the last spot rate.
- Source of data: Bloomberg
- Motivation: high homogeneity among the four countries

5.Data and results: Summary

- Compounding logarithmic returns of data and processing some data analysis

- Parameters estimation for ARMA-GJR models

- Modeling distribution of standardized residuals with a semi-parametric approach:
  - Gaussian kernel for interior of distribution
  - Generalized Pareto Distribution for tails of distribution

- Estimation of copula parameters

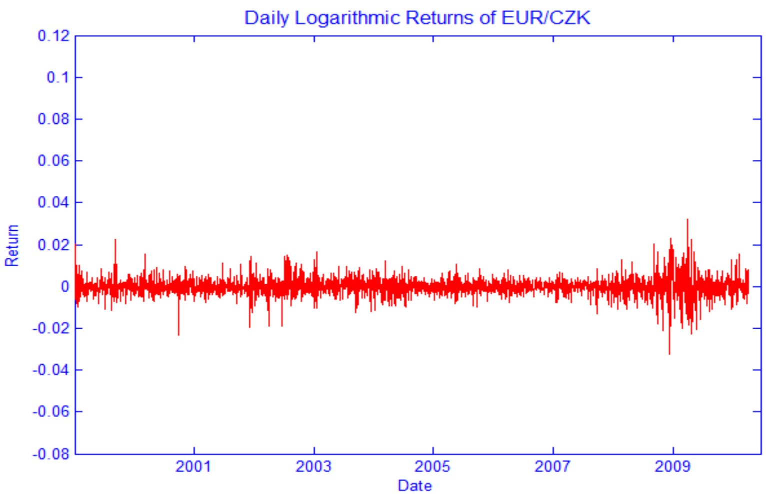
- Simulation of portfolios return distribution using Monte Carlo Simulation

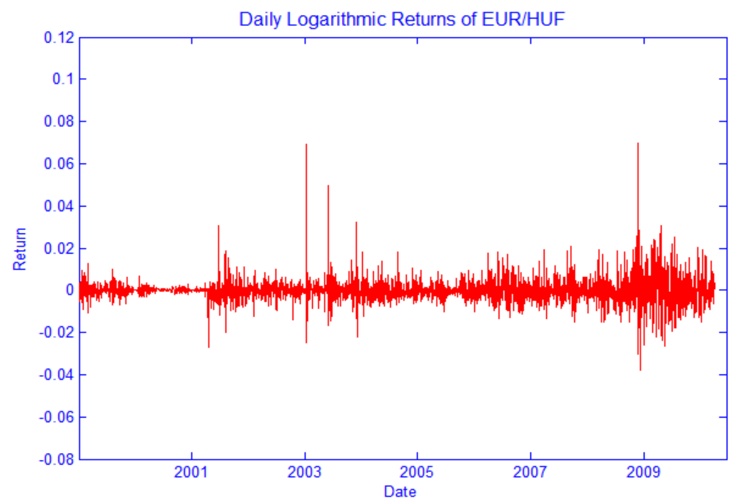
- Risk measures estimation

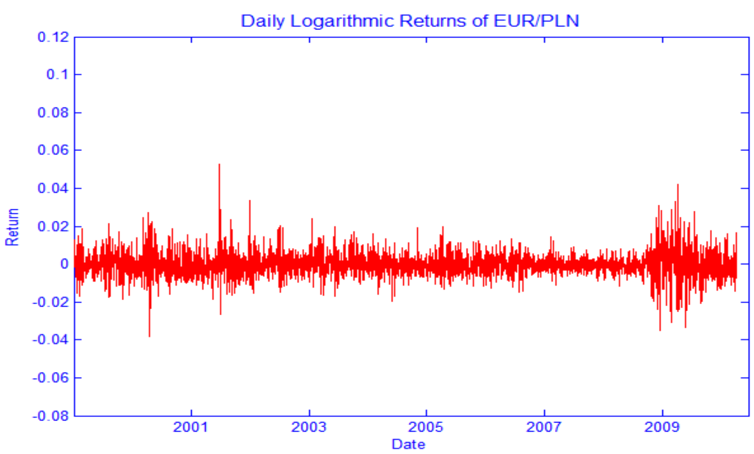
- Backtesting Value-at-Risk models

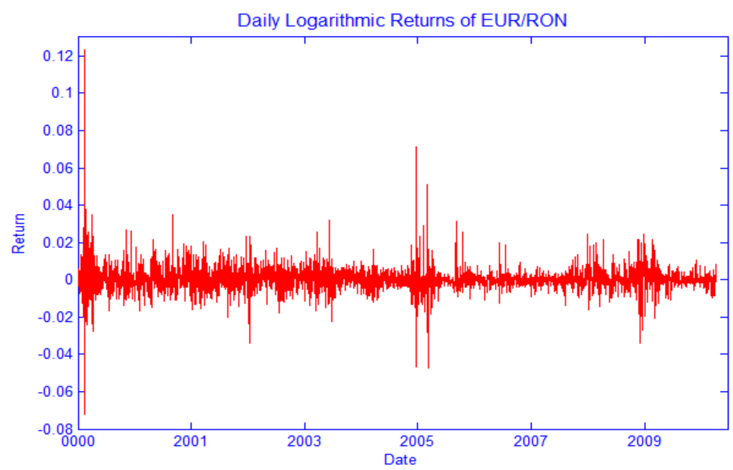
5.Data and results:Exchange rates returns

<u>Facts:</u> skewed, leptokurtic, volatility clusters and heteroskedasticity, autocorrelated, stationary









5. Data and Results: ARMA-GJR estimated parameters

- Positive values of asymmetric term leads to increase of EUR/CZK and EUR/RON volatility
- Asymmetric impact of bad news

		EUR/CZK	EUR/HUF	EUR/PLN	EUR/RON
Constant term	c	-0.0002 (0.0008)	0.0000 (0.4840)	-0.0003 (0.0019)	-0.0035 (0.8650)
AR	ϕ	-0.0729 (0.0001)	0.5361 (0.0000)	-0.0704 (0.0003)	0.9998 (0.0000)
MA	θ	- -	-0.6274 (0.0000)	- -	-0.9964 (0.0000)
Constant term	ω	0.0034 (0.0000)	0.0000 (0.0000)	0.0053 (0.0072)	0.0063 (0.0000)
ARCH	α	0.0790 (0.0000)	0.8765 (0.0000)	0.0873 (0.0000)	0.1398 (0.0000)
GARCH	β	0.9090 (0.0000)	0.1506 (0.0000)	0.9187 (0.0000)	0.8632 (0.0000)
Asymmetric term	γ	0.0160 (0.0041)	-0.0542 (0.0115)	-0.0376 (0.0281)	0.0509 (0.0001)
Student distribution of the errors	DoF	3.8646 (0.0000)	4.2441 (0.0000)	8.0687 (0.0000)	3.5144 (0.0000)

5.Data and results: Ljung-Box Test

- Testing for departures from randomness (autocorrelation and heteroskedasticity) of :
  - standardized residuals
  - squared standardized residuals

Ljung-Box Test for serial correlation

	Standardized Residuals					**Squared Standardized Residuals**			
	EUR/CZK	EUR/HUF	EUR/PLN	EUR/RON		EUR/CZK	EUR/HUF	EUR/PLN	EUR/RON
H	0	0	0	0	H	0	0	0	0
P-value	0.6569	0.6682	0.6856	0.5995	P-value	0.9742	0.3804	0.7708	0.3905
Q-stat	21.6319	21.4343	21.1245	22.6244	Q-stat	13.1829	26.5182	19.534	26.3225
Critical Value	37.6525	37.6525	37.6525	37.6525	Critical Value	37.6525	37.6525	37.6525	37.6525

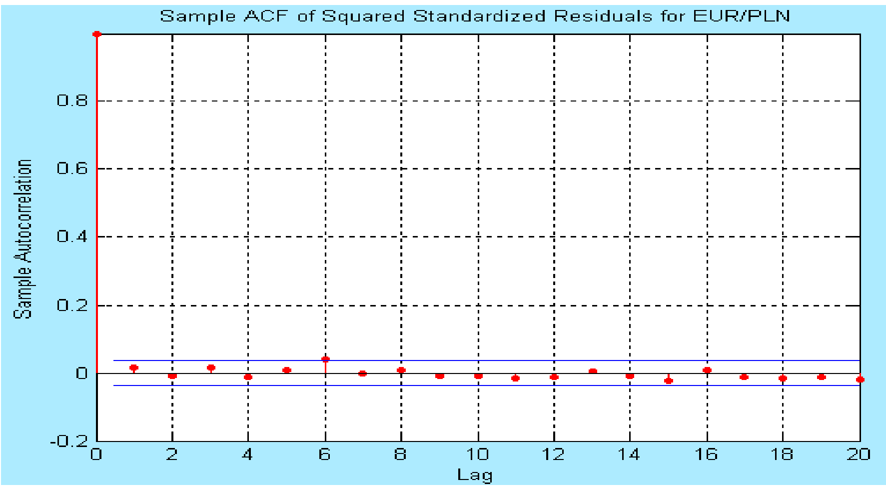
Confidence level: 5%

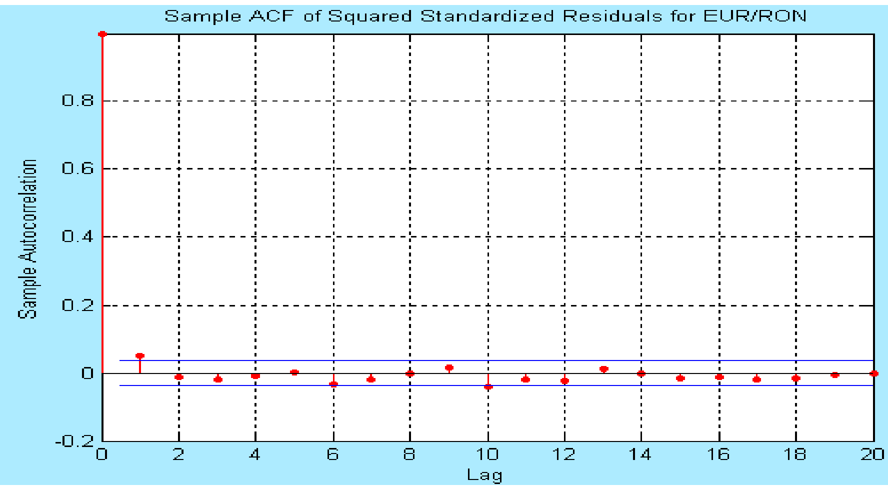
Lag:25

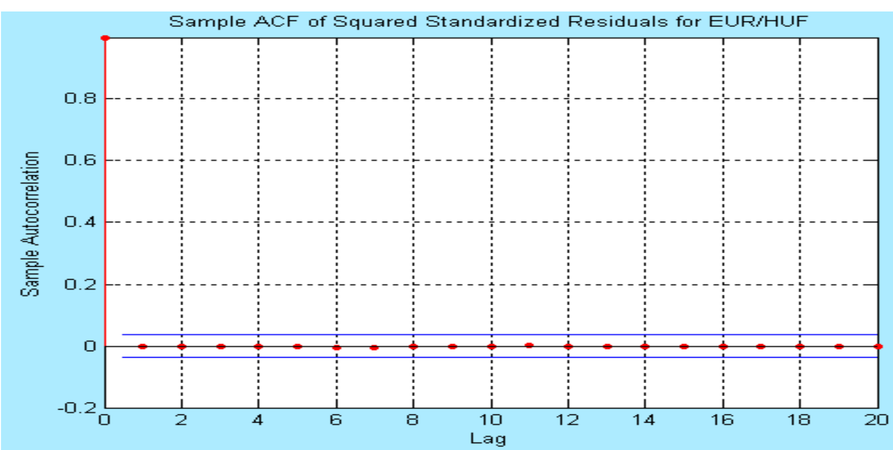
Null Hypothesis: No serial correlation

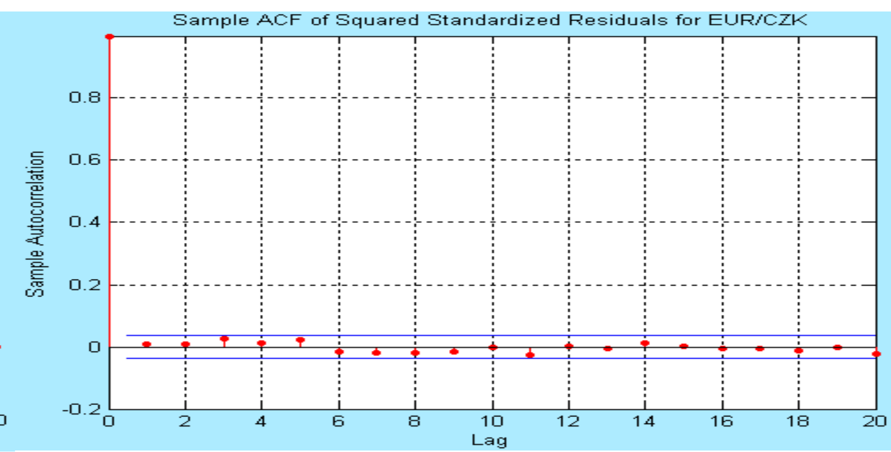
5.Data and results: ACF of Squared Standardized Residuals

- ARMA-GJR Model successfully compensated for autocorrelation and heteroskedasticity





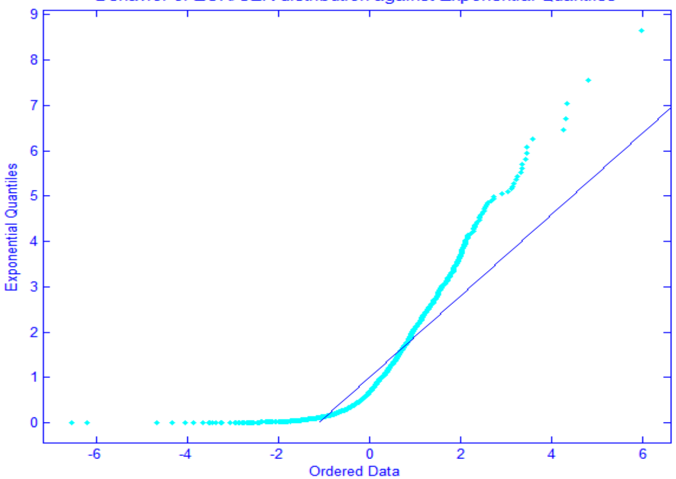




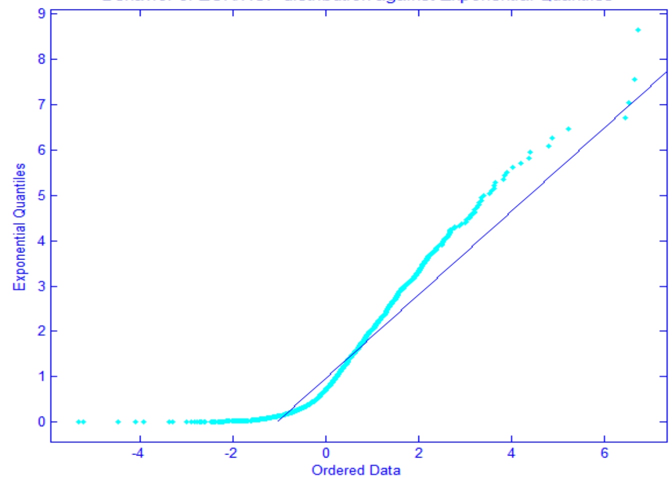
5.Data and results: Preliminary statistics analysis

- As McNeil (1997) suggested the larger the curvature of concave departure (heavy tails) against exponential quantiles the higher the need to use EVT theory

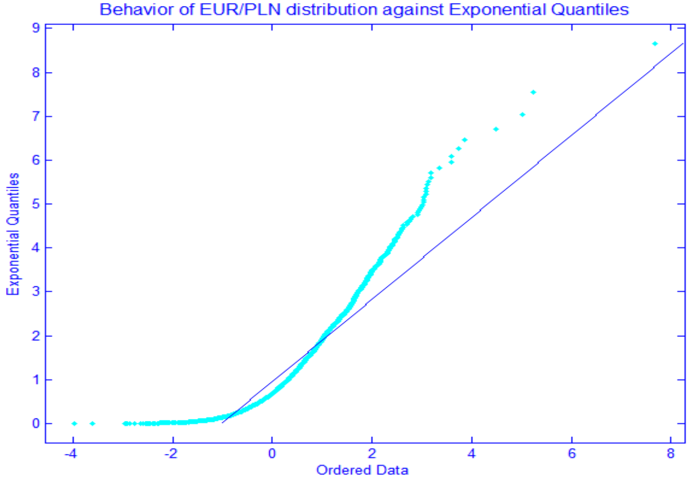
Behavior of EUR/CZK distribution against Exponential Quantiles



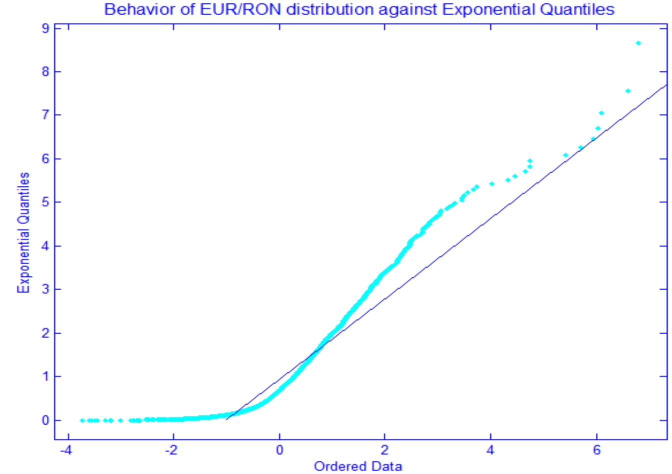
Behavior of EUR/HUF distribution against Exponential Quantiles



Behavior of EUR/PLN distribution against Exponential Quantiles

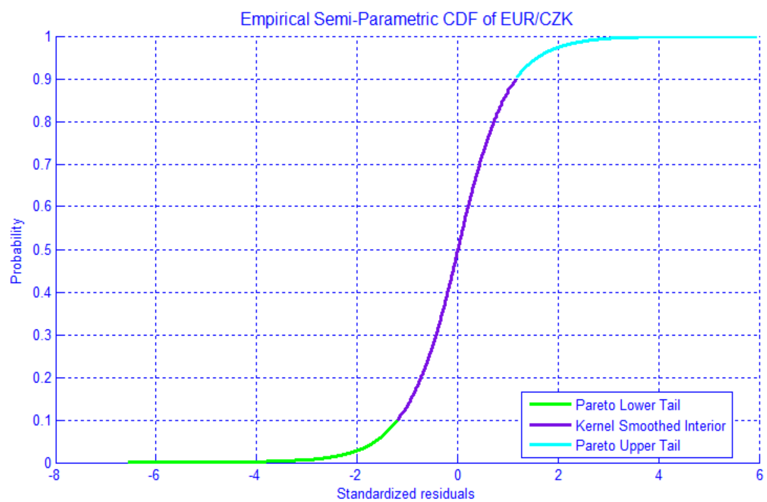


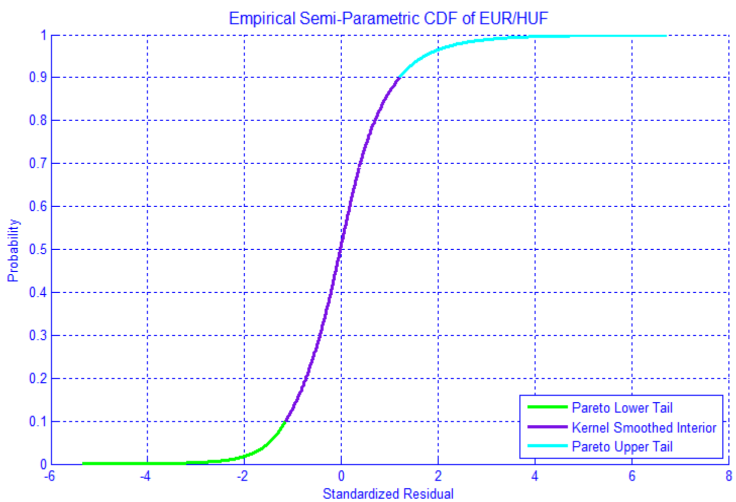
Behavior of EUR/RON distribution against Exponential Quantiles

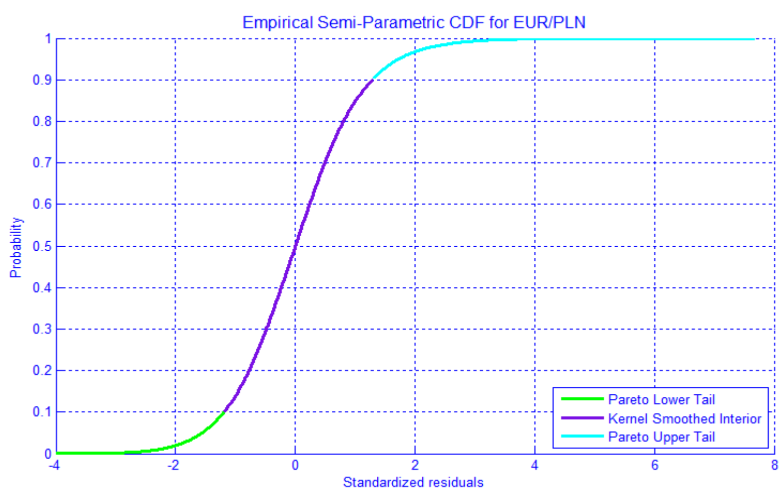


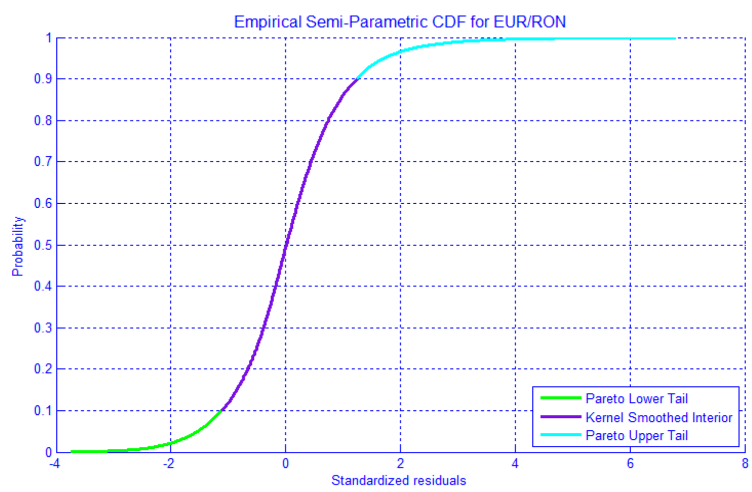
5.Data and Results: Extreme Value Theory application

- Modeling marginal distribution with a semi-parametric approach:
  - Gaussian kernel for interior of distribution
  - Generalized Pareto Distribution for tails









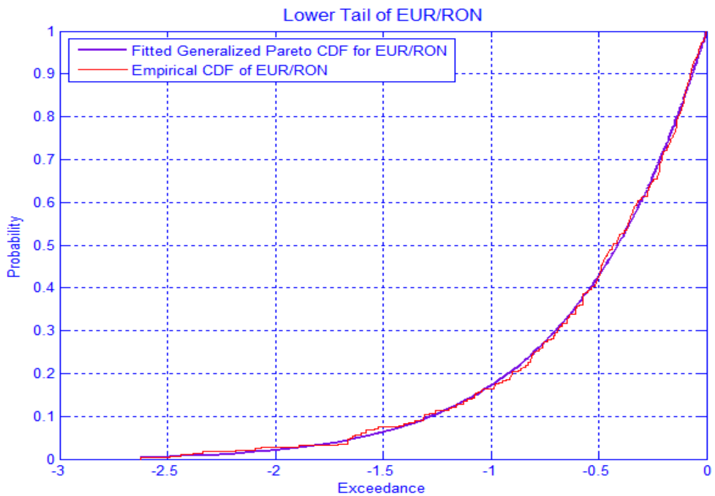
5.Data and Results: Estimation of Generalized Pareto Distribution parameters

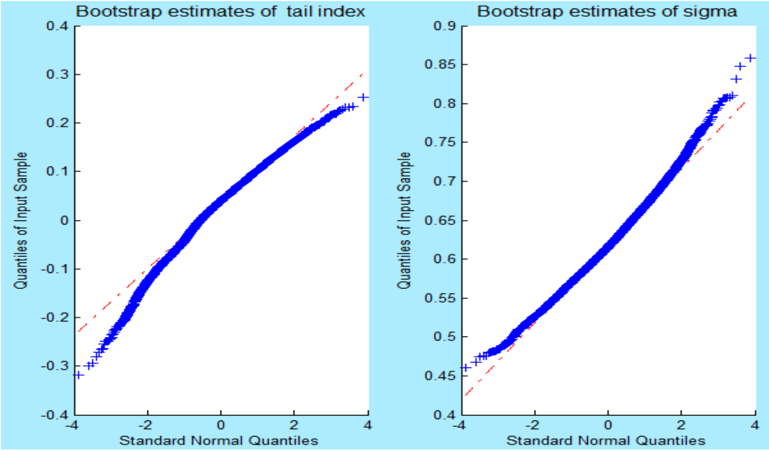
Parameters	EUR/CZK				EUR/HUF			
	Lower tail		Upper tail		Lower tail		Upper tail	
	ξ	σ	ξ	σ	ξ	σ	ξ	σ
ML estimates	0.0813 (0.1797)	0.5934 (0.0000)	0.0140 (0.7998)	0.6307 (0.0000)	0.1264 (0.0452)	0.4327 (0.0000)	0.1253 (0.0698)	0.7151 (0.0000)
Standard Errors	0.0606	0.0507	0.0552	0.0518	0.0631	0.0373	0.0691	0.0652
Lower limits of Confidence interval	-0.0375	0.5019	-0.0941	0.537	0.0027	0.3654	-0.0102	0.5981
Upper limits of Confidence interval	0.2	0.7016	0.1222	0.7407	0.2502	0.5124	0.2607	0.8549

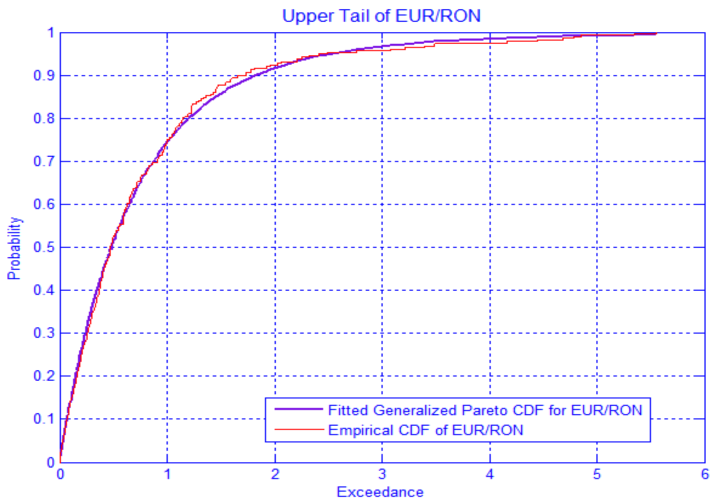
Parameters	EUR/PLN				EUR/RON			
	Lower tail		Upper tail		Lower tail		Upper tail	
	ξ	σ	ξ	σ	ξ	σ	ξ	σ
ML estimates	-0.1017 (0.0372)	0.5328 (0.0000)	0.0495 (0.3503)	0.6099 (0.0000)	-0.0941 (0.0845)	0.6138 (0.0000)	0.1599 (0.0192)	0.6562 (0.0000)
Standard Errors	0.0488	0.0416	0.053	0.0486	0.0564	0.0521	0.0683	0.0593
Lower limits of Confidence interval	-0.1974	0.4571	-0.0544	0.5216	-0.2086	0.5198	0.0261	0.5496
Upper limits of Confidence interval	-0.0059	0.621	0.1534	0.713	0.0283	0.7248	0.2936	0.7835

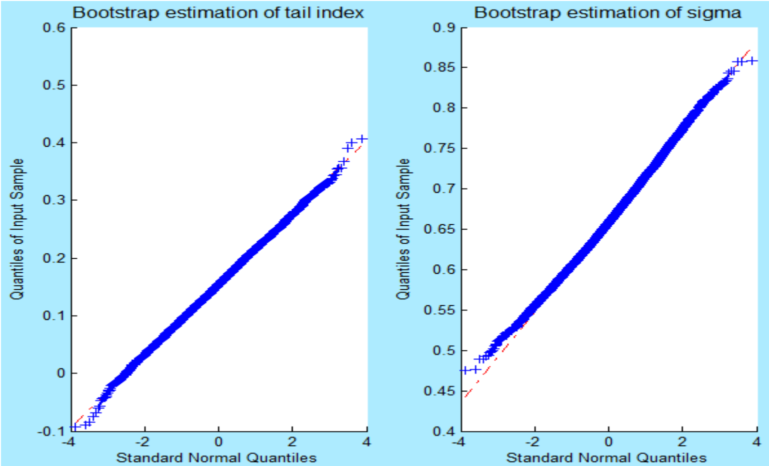
5. Data and Results: Assessing the GPD fit

- GPD approach provides a good fit for tails' distribution









5.Data and Results: Copula parameters for large portfolio

- Canonical Maximum Likelihood estimation: $\hat{\theta}_{CML} = \arg\max \sum_{t=1}^{T} \ln c(\hat{u}_1^t, \dots, \hat{u}_n^t)$
- Positive correlation among the four exchange rates from CEE
- Each currency posts the highest correlation with EUR/PLN and lowest with EUR/RON
- Higher correlation coefficients resulted from T-copula estimation
- Asymmetric tail dependence

DoF	DoF CI	
17.3080	12.1811	22.4348

Correlation Matrix using T-Copula

	EUR/CZK	EUR/HUF	EUR/PLN	EUR/RON
EUR/CZK	1.0000	0.2954	0.3446	0.1453
EUR/HUF	0.2954	1.0000	0.4764	0.2332
EUR/PLN	0.3446	0.4764	1.0000	0.3388
EUR/RON	0.1453	0.2332	0.3388	1.0000

Correlation Matrix using Gaussian-Copula

	EUR/CZK	EUR/HUF	EUR/PLN	EUR/RON
EUR/CZK	1.0000	0.2816	0.3303	0.1345
EUR/HUF	0.2816	1.0000	0.4618	0.2240
EUR/PLN	0.3303	0.4618	1.0000	0.3311
EUR/RON	0.1345	0.2240	0.3311	1.0000

Conditional Dependence with Canonical Vine Copula

Pair	Clayton	SJC	
		Upper tail	Lower tail
EUR/PLN-EUR/CZK	0.1144	0.1403	0.0742
EUR/PLN-EUR/HUF	0.1462	0.1735	0.1371
EUR/PLN-EUR/RON	0.0547	0.0219	0.0102
EUR/CZK-EUR/HUF|EUR/PLN	0.1789	0.2774	0.1286
EUR/CZK-EUR/RON|EUR/PLN	0.0801	0.0844	0.0183
EUR/HUF-EUR/RON|EURPLN,EUR/CZK	0.1072	0.1049	0.0566

5.Data and Results: Copula parameters for EUR/PLN-EUR/RON sub-portfolio

- Differences between parameters estimated with Gumbel and Rotated Gumbel copulas and between SJC tails attest the evidence of asymmetric dependence

Kendall's tau	Theoretical Rho of the sample	Gaussian	T-copula				Clayton			Frank		
		R	R	DoF	CI		θ	CI		θ	CI	
0.2238	0.3443	0.3313	0.3440	16.2281	5.5441	26.9121	0.3815	0.3304	0.4327	2.1826	1.9571	2.4081

Gumbel			Rotated Clayton			Rotated Gumbel			Plackett			SJC	
θ	CI		θ	CI		θ	CI		θ	CI		τ-Lower	τ-Upper
1.2589	1.2256	1.2923	0.4225	0.3713	0.4737	1.2481	1.2147	1.2815	2.9357	2.7102	3.1611	0.1181	0.1884

Time-varying Rotated Gumbel			Time-varying Gumbel			Time-varying SJC					
Ω	β	α	Ω	β	α	Ω-Lower	β-Lower	α-Lower	Ω-Upper	β-Upper	α-Upper
0.9591	-0.0755	-1.4112	-0.1557	0.6135	-0.4331	1.3151	-8.4214	-3.5242	-0.0334	-9.0312	1.5326

5. Data and Results: Tail Dependence and Information Criteria for EUR/PLN-EUR/RON sub-portfolio

- Plackett and Frank copulas recorded the lowest negative log-likelihood values

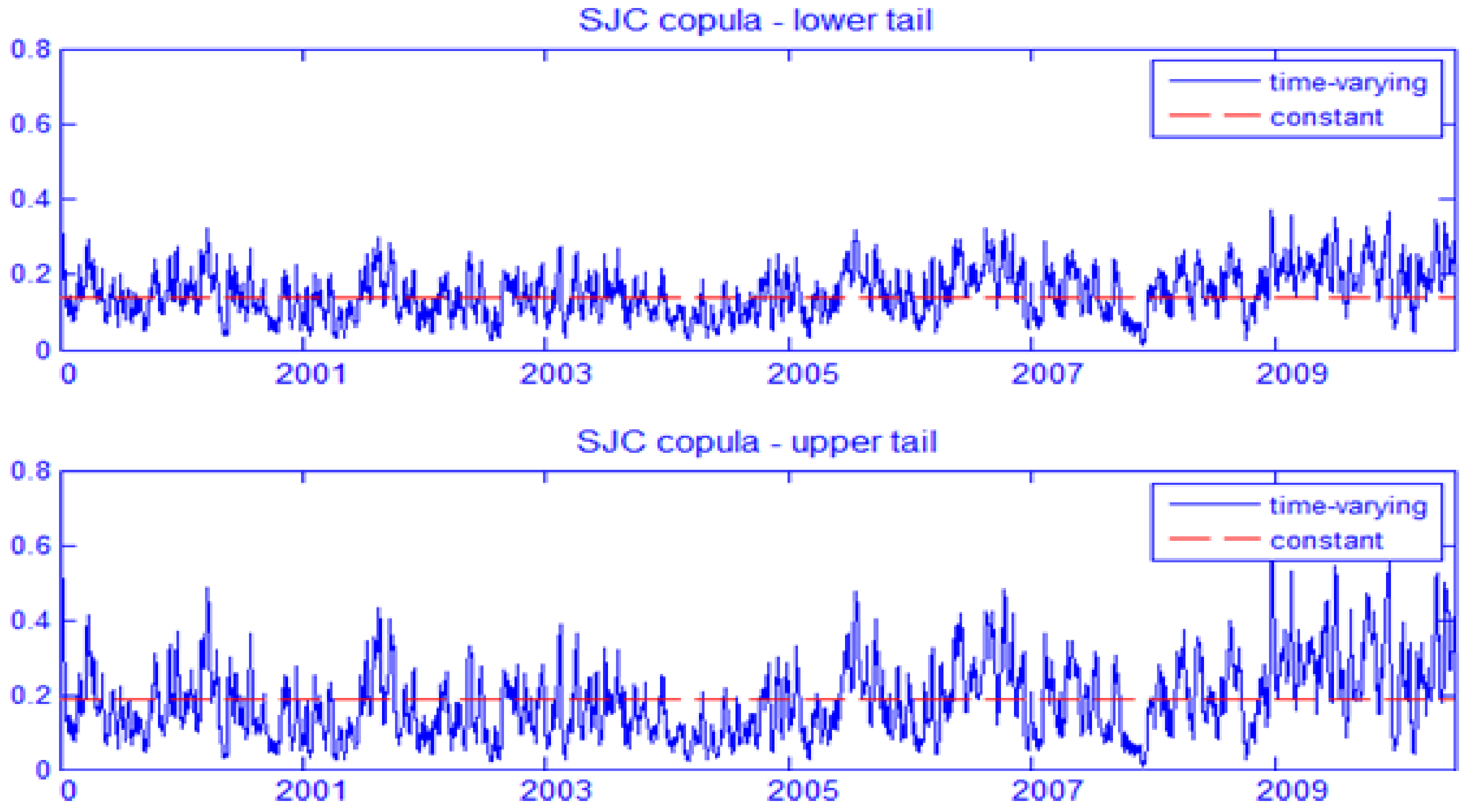
	Tail Dependence	
Copula	**Lower**	**Upper**
Gaussian	0	0
Clayton	0.1627	0
Rotated Clayton	0	0.1939
Plackett	0	0
Frank	0	0
Gumbel	0	0.2657
Rotated Gumbel	0.2574	0
T	0.0099	0.0099
SJC	0.1181	0.1884

		Information Criteria	
Copula	**NLL**	**AIC**	**BIC**
Gaussian	-166.8175	-333.634	-333.632
Clayton	-112.2710	-224.541	-224.539
Rotated Clayton	-138.9871	-277.974	-277.971
Plackett	**-172.1960**	**-344.391**	**-344.389**
Frank	**-166.1478**	**-332.295**	**-332.293**
Gumbel	-159.1701	-318.339	-318.337
Rotated Gumbel	-142.3737	-284.747	-284.745
T	-171.8631	-343.725	-343.721
SJC	-163.9774	-327.953	-327.949
Copula-GARCH			
Gumbel	-172.0255	-344.049	-344.043
Rotated Gumbel	-156.7138	-313.426	-313.419
Symmetrised Joe-Clayton	**-176.9928**	**-353.981**	**-353.969**

5. Data and Results: Regime Switches of tail dependence with Symmetrized Joe-Clayton Copula-GARCH Model

EUR/PLN-EUR/CZK

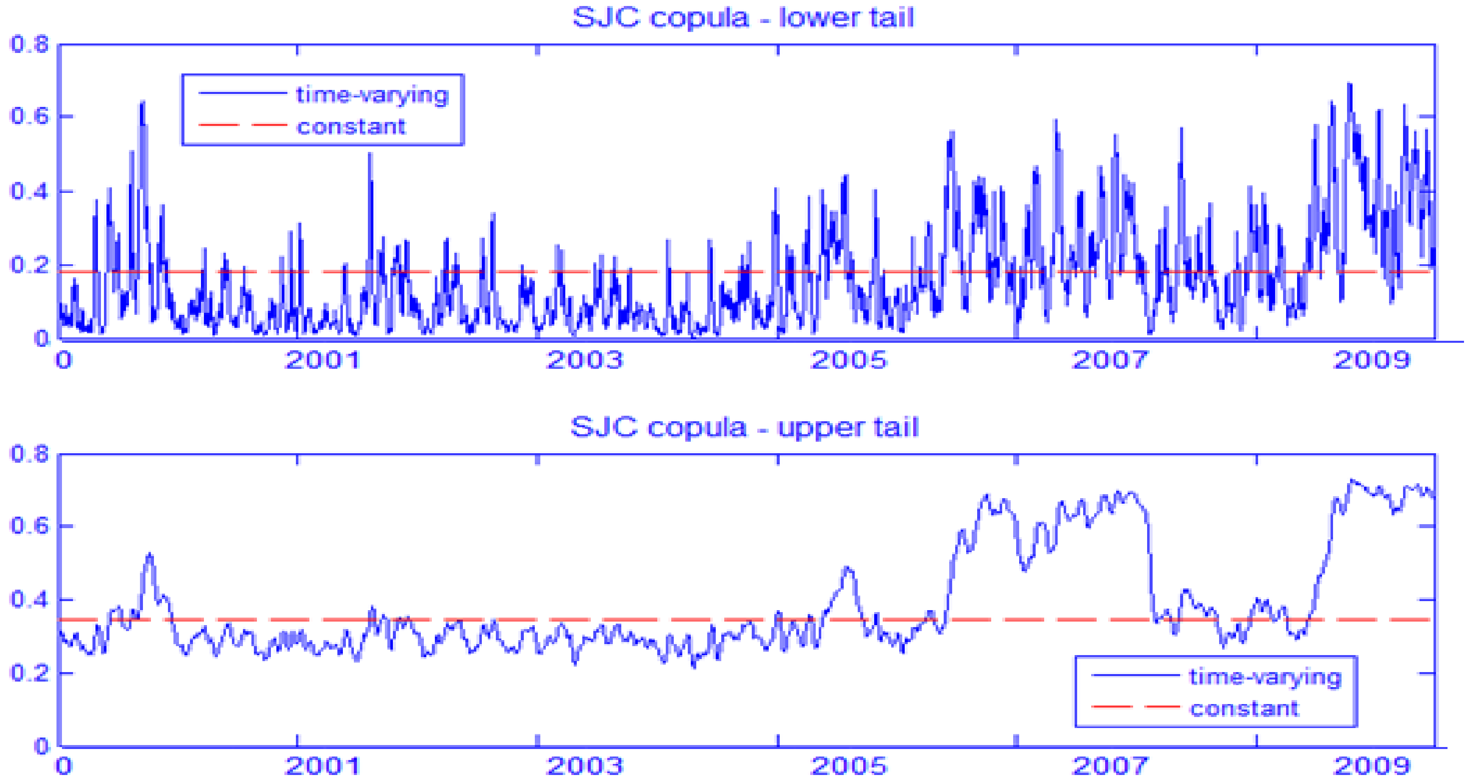
- Low asymmetry between dynamics of tail dependence



5.Data and Results: Regime Switches of tail dependence with Symmetrized Joe-Clayton Copula-GARCH Model

EUR/PLN-EUR/HUF

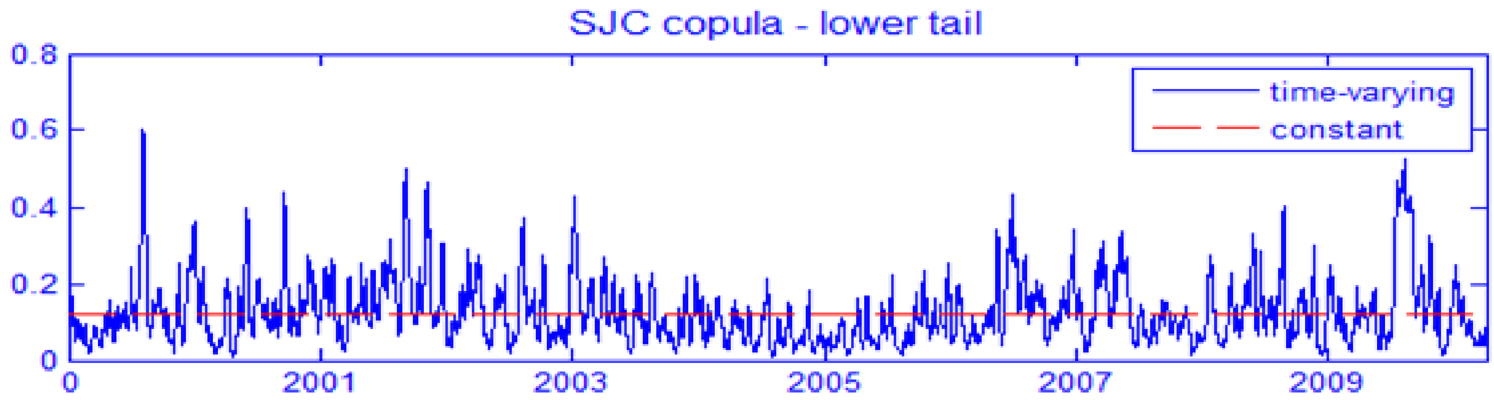
• High dependence in the right tail with beginning of financial crisis
•Markov-Switching regressions for EUR/HUF revealed a suddenly rigidity in transition between states ranging the end of 2006 and the begin of 2007

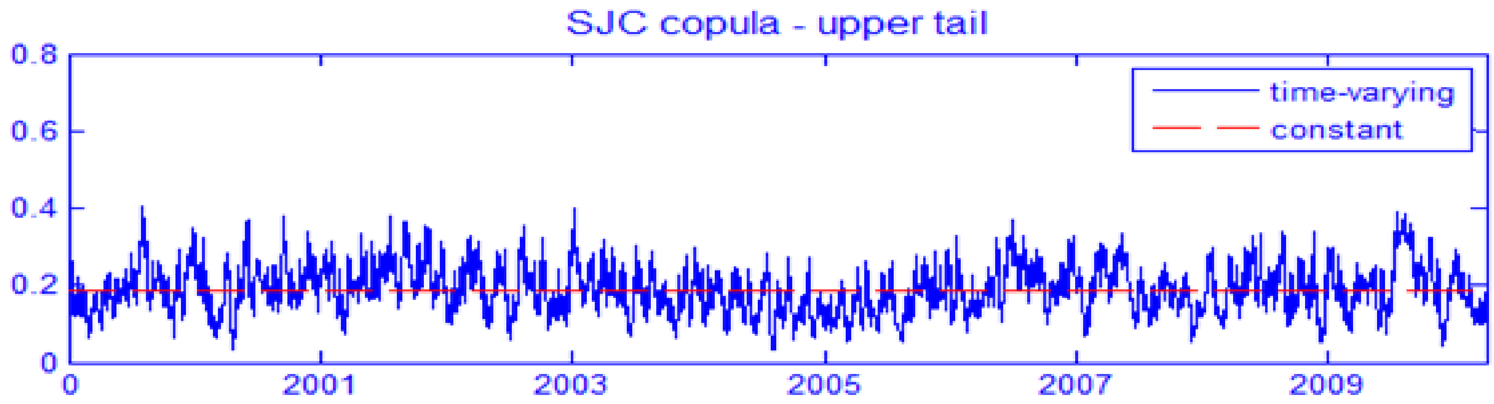


5.Data and Results: Regime Switches of tail dependence with Symmetrized Joe-Clayton Copula-GARCH Model

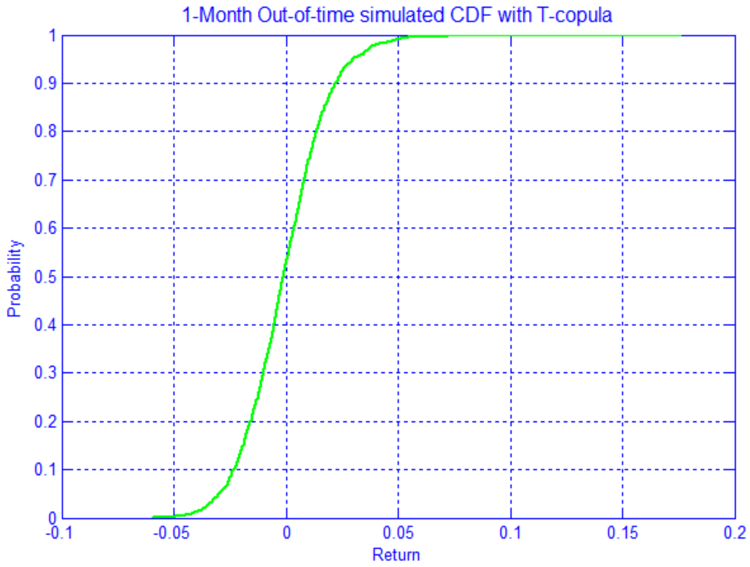
EUR/PLN-EUR/RON

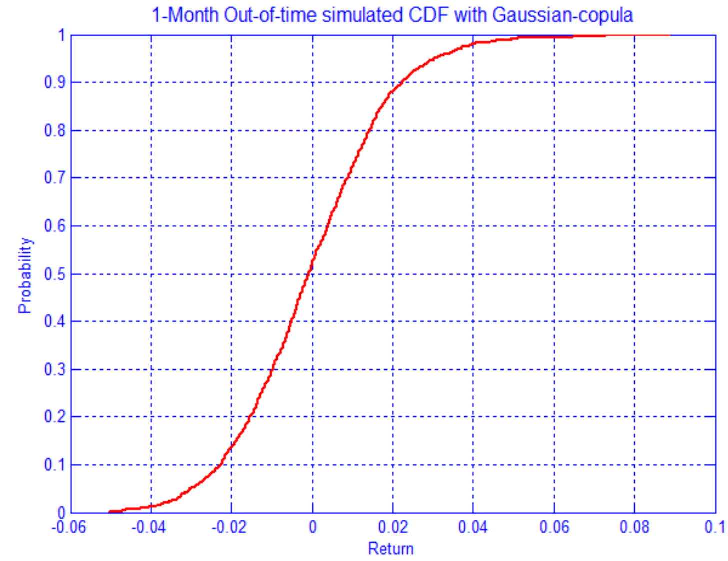
- Right asymmetric tail dependence
- Switches of upper tail seem very noisy

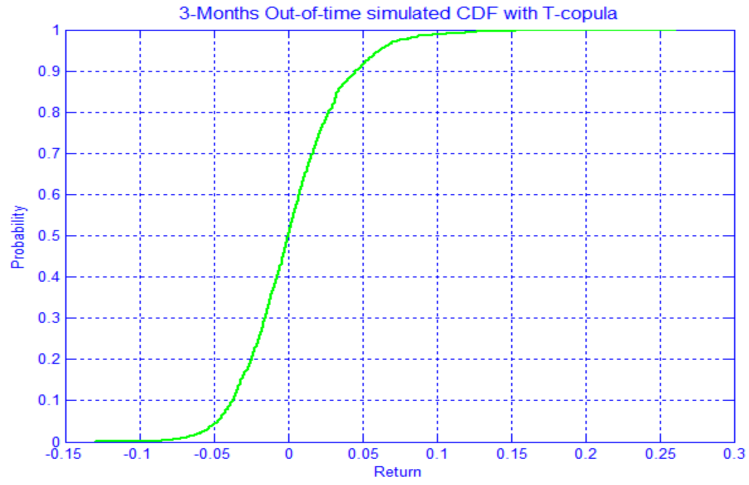


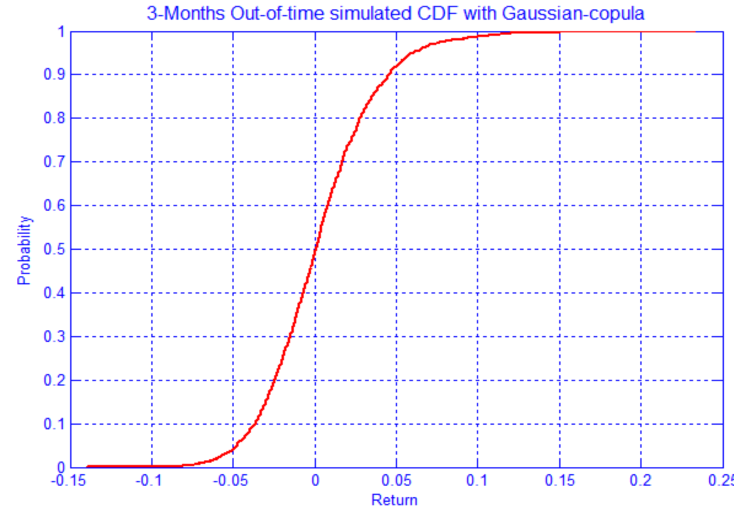


5.Data and Results:Monte Carlo simulation of cumulative distribution for large portfolio returns





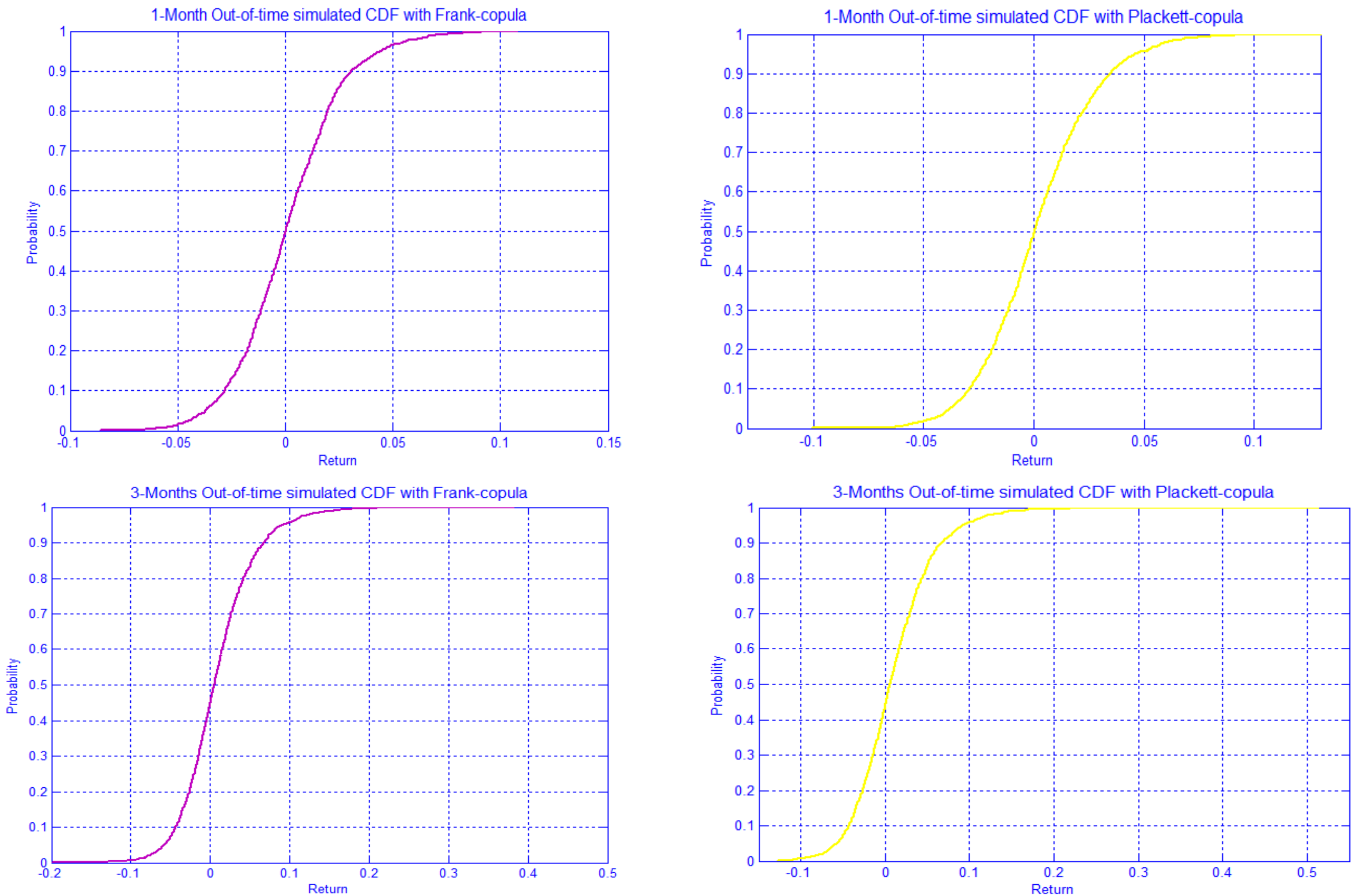




5.Data and Results: Estimation of VaR and CVaR for large portfolio

Horizon5	Quartile		T-copula VaR	Gaussian-Copula VaR		T-copula VaR	Gaussian-Copula VaR	Min. and max. empirical return	Out-of-Time realized return
1 day	0.05	VaR	-0.7672	-0.7458	CVaR	-0.9694	-0.9368	-2.4476	**0.1997**
	0.01		-1.0586	-1.0424		-1.2604	-1.2104		
	0.95		0.6942	0.6876		1.0186	0.9724	2.9239	
	0.99		1.1610	1.0788		1.6193	1.4892		
5 days	0.05	VaR	-1.5504	-1.5285	CVaR	-1.9738	-1.9305	-5.3024	**-1.2347**
	0.01		-2.1861	-2.1500		-2.7503	-2.6303		
	0.95		1.6023	1.5935		2.1388	2.1383	6.4693	
	0.99		2.4294	2.4947		2.8754	3.0795		
10 days	0.05	VaR	-2.0100	-1.9870	CVaR	-2.5471	-2.5164	-5.6800	**-1.3248**
	0.01		-2.9286	-2.8571		-3.3710	-3.3767		
	0.95		2.2270	2.2400		3.1834	3.0859	7.2602	
	0.99		3.6803	3.5362		4.9917	4.4554		
1 month	0.05	VaR	-3.0695	-3.1148	CVaR	-3.8660	-3.8056	-5.1677	**-2.9442**
	0.01		-4.2903	-4.3055		-5.0748	-4.8053		
	0.95		3.3703	3.4187		4.8860	4.8636	9.1040	
	0.99		5.6392	5.5865		7.5849	7.4142		
3 months	0.05	VaR	-4.9829	-5.0366	CVaR	-6.4952	-6.3666	-9.2584	**1.2677**
	0.01		-7.4981	-7.2513		-8.7605	-8.4968		
	0.95		6.8738	6.8910		10.0183	10.0590	18.7701	
	0.99		11.9919	11.9431		16.3543	15.5711		

5.Data and Results:Monte Carlo simulation of cumulative distribution for EUR/PLN-EUR/RON sub-portfolio

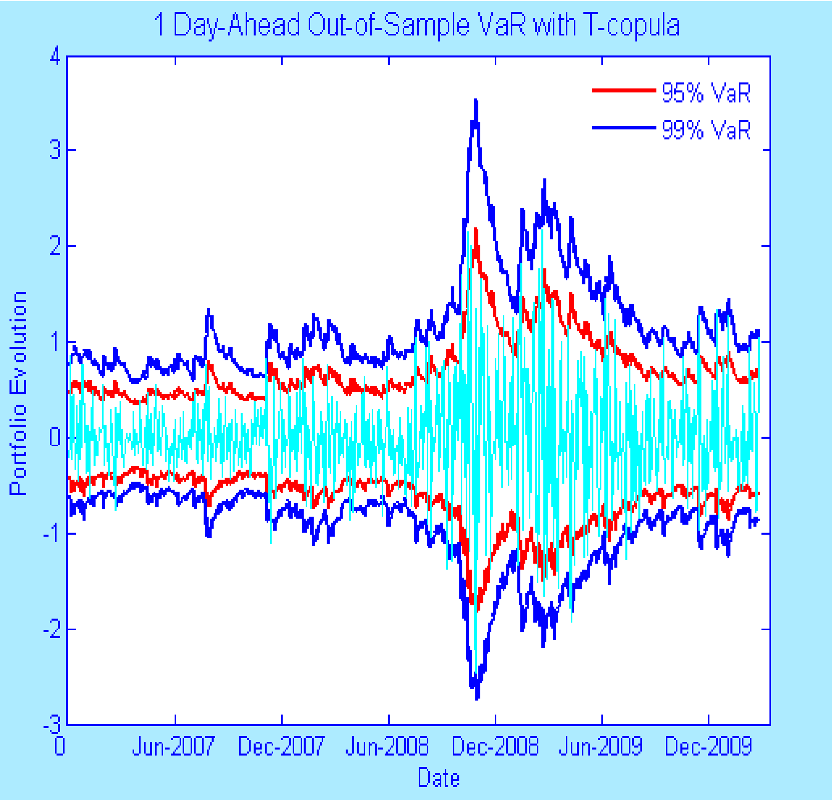


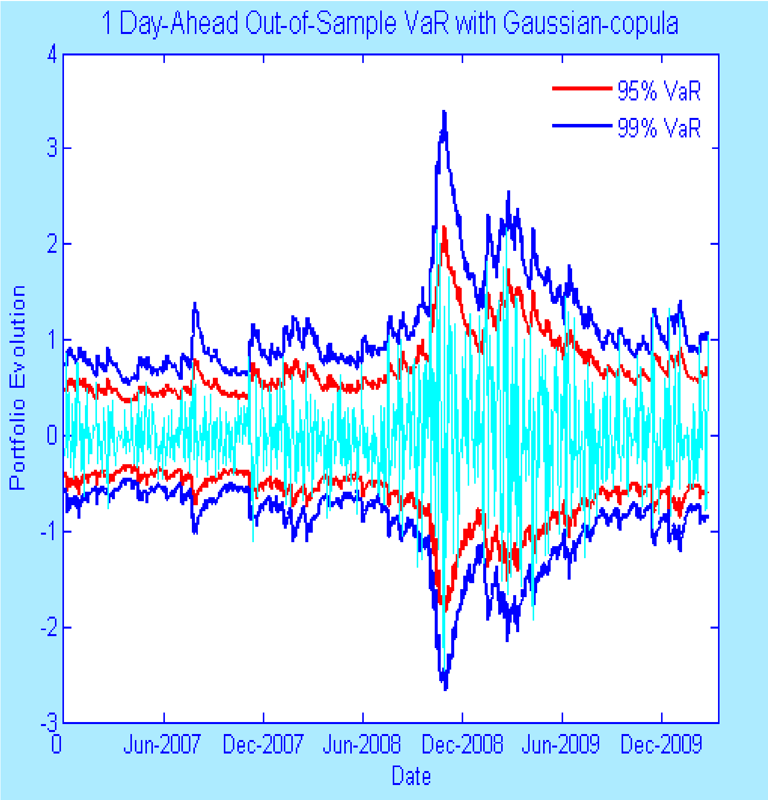
5.Data and Results: Estimation of VaR and CVaR for EUR/PLN-EUR/RON sub-portfolio

Horizon	Quartile		Frank-copula VaR	Plackett-Copula VaR		Frank-copula VaR	Plackett-Copula VaR	Min. and max. empirical return	Out-of-Time realized return
1 day	0.05	VaR	-0.9485	-0.9936	CVaR	-1.2759	-1.3022	-4.0511	**0.4116**
	0.01		-1.4261	-1.4874		-1.7752	-1.8018		
	0.95		0.9036	0.9167		1.2445	1.3030	5.8615	
	0.99		1.4666	1.5054		1.7806	2.0016		
5 days	0.05	VaR	-2.0450	-2.0300	CVaR	-2.7283	-2.6029	-4.8666	**-1.1141**
	0.01		-3.2014	-2.9115		-3.7722	-3.4631		
	0.95		1.8590	1.8269		2.5676	2.5472	6.5516	
	0.99		2.9923	2.9373		3.7096	3.7520		
10 days	0.05	VaR	-2.8445	-2.8308	CVaR	-3.7579	-3.5763	-5.5519	**-1.0764**
	0.01		-4.1471	-4.2037		-5.2994	-4.6958		
	0.95		2.7004	2.9144		3.7518	3.9300	7.5991	
	0.99		4.4391	4.5338		5.3432	5.4369		
1 month	0.05	VaR	-4.4102	-4.3280	CVaR	-5.6651	-5.5022	-5.5050	**-3.1584**
	0.01		-6.2009	-6.1906		-7.6919	-7.2893		
	0.95		4.1188	4.5157		5.7739	6.3479	9.6777	
	0.99		6.7624	7.1696		7.9586	9.6292		
3 months	0.05	VaR	-7.7137	-7.4042	CVaR	-10.2423	-9.8925	-10.2125	**1.9176**
	0.01		-11.7625	-10.9088		-14.9252	-14.4789		
	0.95		7.5080	8.6671		11.7263	13.1176	22.1169	
	0.99		13.2778	15.5488		18.1708	21.1178		

5. Data and Results: Out-of-Sample VaR for large portfolio

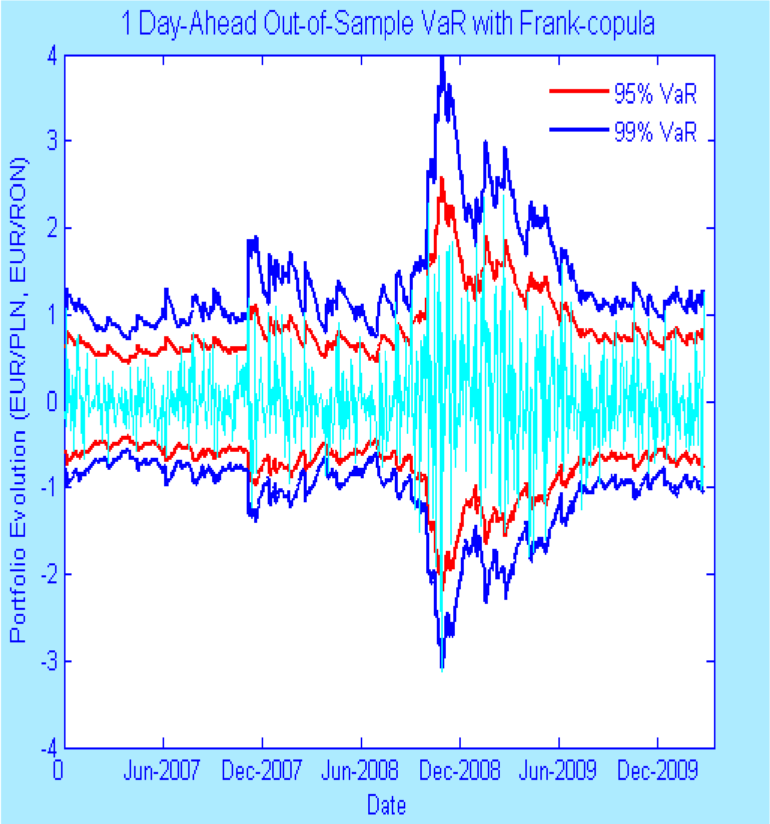
- Rolling window method
- 1 day window length
- Estimation sample :2062 observations
- Forecasting sample: last 3 years of the sample, 808 observations

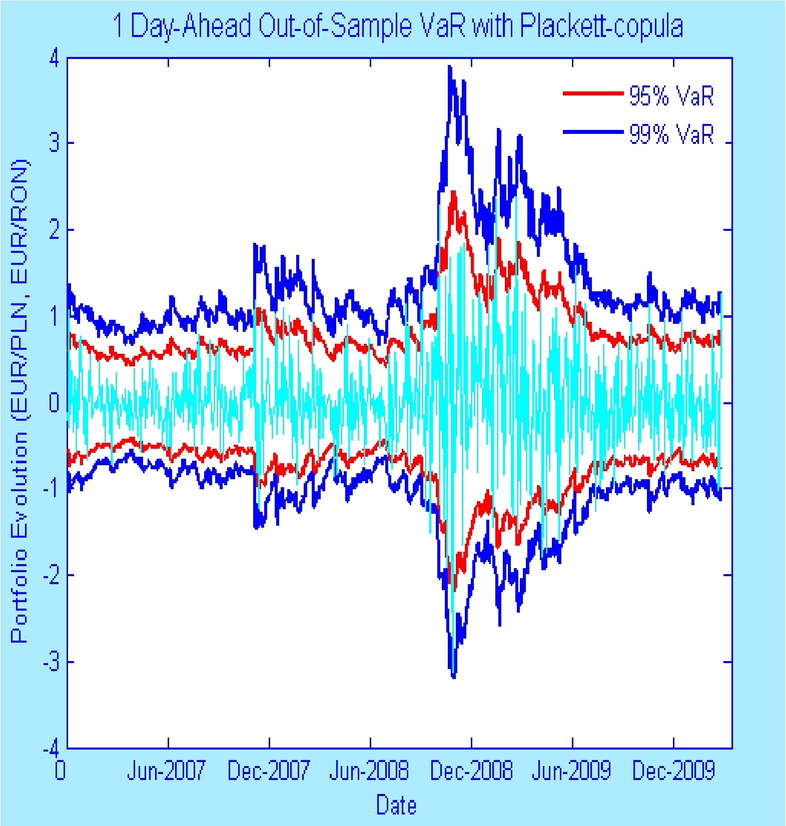




5. Data and Results: Out-of-Sample VaR for EUR/PLN-EUR/RON sub-portfolio

- Rolling window method
- 1 day window length
- Estimation sample :2062 observations
- Forecasting sample: last 3 years of the sample, 808 observations





5. Data and Results: Backtesting Out-of-Sample results

•Null hypothesis of Bernoulli test: VaR model is accurate

•Null hypothesis of Kupiec test: Indicator function is accurate in levelling the significance level of VaR

Bernoulli Backtest and Calibration to Basel II *Traffic light*

	Copula	95% VaR		99% VaR	
		0.05	0.95	0.01	0.99
Large Portfolio	T	🟡 7.17%	🟡 7.29%	🟢 1.36%	🟢 1.48%
	Gaussian	🟡 7.42%	🟡 7.17%	🟢 1.61%	🟡 1.98%
EUR/PLN-EUR/CZK Portfolio	T	🟢 5.69%	🟢 5.69%	🟢 1.48%	🟡 2.35%
	Gumbel	🟢 5.69%	🟡 7.05%	🟢 1.36%	🟡 1.98%
EUR/PLN-EUR/HUF Portfolio	T	🟢 5.32%	🟡 7.42%	🟢 1.11%	🟢 1.48%
	Gumbel	🟢 5.81%	🟡 8.41%	🟢 1.36%	🟢 1.24%
EUR/PLN-EUR/RON Portfolio	Frank	🟢 **4.45%**	🟢 **4.94%**	🟢 1.48%	🟢 1.11%
	Plackett	🟢 **4.57%**	🟢 5.19%	🟢 1.36%	🟢 1.36%

Kupiec Backtest

	Copula	95% VaR		99% VaR	
		0.05	0.95	0.01	0.99
Large Portfolio	T	7.0987	7.8832	0.9493***	1.6597***
	Gaussian	8.7042	7.0987	2.5394***	6.0736*
EUR/PLN-EUR/CZK Portfolio	T	0.7681	0.7681***	1.6597***	10.7608
	Gumbel	0.7681***	6.3514*	0.9493***	6.0736*
EUR/PLN-EUR/HUF Portfolio	T	0.1657***	8.7042	0.0996***	1.6597***
	Gumbel	1.0623***	16.5231	0.9493***	0.4232***
EUR/PLN-EUR/RON Portfolio	Frank	0.3181***	0.0053***	1.6597***	0.0996***
	Plackett	0.3181***	0.0617***	0.9493***	0.9493***

***Denotes the acceptance of null at 10%; χ-squared critical value = 2.7055
**Denotes the acceptance of null at 5%; χ-squared critical value = 3.8415
*Denotes the acceptance of null at 1%; χ-squared critical value = 6.6349

6. Conclusions

- ARMA-GJR models performed well in order to compensate for autocorrelation and heteroskedasticity

- GPD approach provided a good fit for tail's parameters estimation

- Canonical Vine and copula-GARCH models revealed an asymmetric dependence between periods of appreciation and depreciation

- Backtesting results showed that:

  - Plackett and Frank copulas performs well for the middle range of the sample

  - Gaussian copula performs poorly in out-of-sample forecasting of VaR due to its structure of no tail dependence

  - Gumbel and Student copulas provide satisfactory results

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Thank you!